The following is a review of the Risk Management Applications of Derivatives principles designed to address the learning outcome statements set forth by CFA Institute. Cross-Reference to CFA Institute Assigned Reading #26.

**RISK MANAGEMENT APPLICATIONS OF FORWARD AND FUTURES STRATEGIES**

**Study Session 15**

**Exam Focus**

Be able to perform any of the calculations using futures contracts to alter the beta of an equity portfolio, alter the duration of a bond portfolio, change the portfolio allocation among various classes of debt and equity, create synthetic positions, or preinvest an expected cash flow. Understand the basic concepts and risks involved.

**Warm-Up: Futures and Forwards**

The CFA curriculum includes a long section of optional material that has no Learning Outcome Statements and will not be tested directly. It does review how to calculate the number of contracts to modify duration of a fixed income position and you are responsible for those calculations. The formulas are consistent with those covered earlier in the fixed income lesson but are laid out in a different form. We will cover the calculations in this reading assignment and review the formulas needed.

Forward and futures contracts are effective tools for managing both interest rate and equity risks. Although very similar, however, one or the other may be preferred in some cases. The primary differences between the two are that forward contracts can be tailored to meet the specific needs of the counterparties but have higher default risk and less liquidity than futures. In contrast, futures contracts are standardized, so they are less likely to be exactly what the two parties need; however, they trade on an exchange, so the risk of loss from default is minimal.

**Adjusting the Portfolio Beta**

**LOS 26.a: Demonstrate the use of equity futures contracts to achieve a target beta for a stock portfolio and calculate and interpret the number of futures contracts required.**

To modify the beta of an equity portfolio with futures on an equity index, we need to know the beta of the equity portfolio to be hedged or leveraged, as well as the beta of the futures contract. Both betas would be measured with respect to the reference index.

You might ask, “Shouldn’t the beta of the index futures contract equal one?” The answer is no, for two reasons. First, for an index like the S&P 500, it will probably be close to
one, but for a more precise hedge, a manager should compute the beta. Second, as seen later, we may wish to adjust exposure with respect to a class of equity (e.g., small-cap stocks) where the beta will be very different from one.

Recall the formula for beta:

\[
\beta_i = \frac{\text{Cov}(i,m)}{\sigma_m^2}
\]

where:
- \(i\) = an individual stock, equity portfolio, or equity index
- \(\text{Cov}(i,m)\) = covariance of returns on asset \(i\) with the market
- \(\sigma_m^2\) = variance of the market returns

Having computed our betas and selected a target beta, we can find the appropriate number of contracts to sell or buy to hedge or leverage the position (reduce or increase beta), respectively:

\[
\text{number of contracts} = \left(\frac{\beta_T - \beta_P}{\beta_f}\right) \left(\frac{V_p}{P_f \text{ (multiplier)}}\right)
\]

where:
- \(\beta_T\) = desired portfolio beta
- \(\beta_P\) = portfolio beta
- \(\beta_f\) = equity futures contract beta
- \(V_p\) = current value of the portfolio
- \(P_f\) = futures price

Professor’s Note: If you recall the earlier fixed income hedging formula you should recognize this is essentially the same formula but using beta instead of duration as the risk measure. In addition, there is no yield beta for stock.

Example: Adjusting portfolio beta

A manager of a $5,000,000 portfolio wants to increase the beta from the current value of 0.8 to 1.1. The beta on the futures contract is 1.05, and the total futures price is $240,000.

Calculate the required number of futures contracts to achieve a beta of 1.1.

Calculate the required number of futures contracts to achieve a beta of 0.0.
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #26 – Risk Management Applications of Forward and Futures Strategies

Answer:

\[
\text{target beta} = 1.1 \\
\text{number of contracts} = \left( \frac{1.1 - 0.8}{1.05} \right) \left( \frac{\$5,000,000}{\$240,000} \right) = 5.95, \text{ buy 6 contracts at $240,000}
\]

Answer:

\[
\text{target beta} = 0.0 \\
\text{number of contracts} = \left( \frac{0 - 0.8}{1.05} \right) \left( \frac{\$5,000,000}{\$240,000} \right) = -15.87, \text{ sell 16 contracts at $240,000}
\]

**Hedging Is Rarely Perfect**

It is highly unusual for the results of the risk adjustment to be perfect. Generically this is referred to as basis risk. Basis risk occurs whenever the item hedged (in the numerator of the hedge formula calculation) is not a perfect match for the hedging vehicle (in the denominator of the hedge formula) and, as a result, the two change in relationship to each other in unpredictably ways. The typical reasons for basis risk include:

- The numerator and denominator are not based on the same item. For example:
  - A stock portfolio hedged using a contract based on the S&P 500 Index.
  - A bond portfolio hedged with a Treasury bond contract based on a single deliverable Treasury bond.
- The betas and durations used in the hedge calculation do not reflect the actual subsequent market value changes of the portfolio or contract, a very common issue.
- The hedge results are measured prior to contract expiration and/or the hedge is closed prior to contract expiration. Alternatively, the hedge may need to be extended after the expiration of the initial contract position.
  - Note: If a contract is held to expiration, the contract price will converge and be equal to the spot price of the underlying at expiration, a relationship called convergence. This is not basis risk because it is a known change between spot and forward price. Holding contracts to expiration reduces basis risk.
- The number of contracts is rounded.
  - The exam convention is to round 0.5 or greater up to the closest whole number and round less than 0.5 down.
- The future and spot price are not fairly priced based on the cash and carry arbitrage model.

Effective beta of the position can be measured ex post (after the fact) as:

\[
\text{effective beta} = \frac{\% \text{ change in value of the portfolio}}{\% \text{ change in the index}}
\]
Example: Ex Post Results Evaluation

Continuing the previous example, assume the unhedged portfolio increased in value 5.1% from $5,000,000 to $5,255,000, and the futures price also increased 5.1% from 240,000 to 252,240. One month remains to contract expiration. The market had a return of 5.2%. For each scenario, compute the i) hedged portfolio ending value, ii) the ex post beta, and iii) give two relevant reasons the ex post beta was not as expected.

Answers:

Scenario 1, target beta of 1.1 and 6 (not 5.95) contracts purchased:

i) hedged portfolio ending value = unhedged ending value + G/L on contracts

   The contract price increased $12,240 for a gain on the long position of:

   $12,240 × 6 = $73,440

   hedged portfolio ending value = $5,255,000 + 73,440 = $5,328,440

   hedged portfolio return = (5,328,440/5,000,000) – 1 = +6.57%

ii) The effective beta was: 6.57/5.2 = 1.26.

iii)

   • The number of contracts was rounded up, which in a rising market, increased the gain and hedged portfolio percent return and effective beta of the hedged portfolio.
   • The ex post valuation period was not at contract expiration. The relationship of futures and underlying prices can, therefore, change in unexpected ways.
   • The performance of the portfolio and/or index may have been different from their ex ante betas. Given that the portfolio and contract increased by the same percent amount, they acted as if their betas were the same and did not reflect the initial estimates of beta.
Scenario 2, target beta of 0 and 16 (not 15.87) contracts sold:

i) hedged portfolio ending value = unhedged ending value + G/L on contracts

The contract price increased $12,240 for a loss on the short position of:

$12,240 \times 16 = $195,840

hedged portfolio ending value = $5,255,000 – 195,840 = $5,059,160

hedged portfolio return = (5,059,160 / 5,000,000) – 1 = +1.18%

ii) The effective beta was: 1.18 / 5.2 = 0.23.

iii)

- The number of contracts sold was rounded up, which in a rising market, increased the loss on the short position and reduced the percent return on the hedged portfolio and its effective beta.
- The ex post valuation was not at contract expiration. The relationship of futures and underlying prices can, therefore, change in unexpected ways.
- The performance of the portfolio and/or index could have been different from their ex ante betas. Because ex post beta was higher than the target of zero, the portfolio beta must have been higher or futures beta less than expected. The portfolio could have acted as if its beta were more than 0.8 and/or the contract less than 1.05.

**Synthetic Positions**

LOS 26.b: Construct a synthetic stock index fund using cash and stock index futures (equitizing cash).

Synthetic positions are based on the same formulas using beta or duration to modify portfolio risk. However, synthetic positions more precisely replicate the same initial investment and ending results that would have occurred if the replicated position had been owned instead. Synthetic equity or bond positions require purchasing contracts and holding sufficient cash equivalents earning the risk-free rate to pay for the contracts at expiration. Alternatively, synthetic cash positions involve holding the underlying and shorting contracts to hedge the position in such a way that the hedged position “earns” the risk-free rate over the hedging period.
In both cases, the number of contracts is computed using the previous risk modification formulas; however, the quantity to hedge (in the numerator of the hedging formula) is the FV of the amount to modify.

- If the objective is to create synthetic equity from cash and the desired $\beta_T$ is the same as the $\beta_F$, then the first term in the calculation becomes $(\beta_T - 0) / \beta_F = 1.0$. Because it has no effect on the calculation, the betas can be “ignored.”
- If the objective is to create synthetic cash from equity and the existing $\beta_P$ is the same as the $\beta_F$, then the first term in the calculation becomes $(0 - \beta_P) / \beta_F = -1.0$. Because it has no effect on the calculation, the betas can be “ignored.”
- In other cases, the existing or desired betas are not the same as the futures beta and will be given. In such cases, the betas are used and do affect the computation.

The cash equivalents in the synthetic position may be variously referred to as: cash equivalents, a bond or zero coupon bond, a risk-free bond or risk-free zero coupon bond, or any other equivalent terminology.

**Example: Synthetic Positions**

Manager A holds $25,000,000 market value of 3-month Treasury bills yielding 1% and wishes to create $20,000,000 of synthetic S&P 500 stock exposure for three months. The S&P contract is priced at 1,750, the dollar multiplier is 250, and the underlying stocks have a dividend yield of 2.5%.

Calculate the number of contracts to buy or sell and the zero coupon position to take.

**Answer:**

Purchase 46 contracts. (Because no betas were given, it is presumed the desired beta is the same as the futures beta. Purely for illustration, assume they are both 1.07. They will have no affect.)

\[
N_F = \left[\frac{(1.07 - 0)}{1.07}\right] \times \left[\frac{20,000,000 \times 1.01^{3/12}}{1,750 \times 250}\right]
\]

\[
= (1) \times \frac{20,049,814}{437,500} = 45.83 \approx 46
\]

This is a full “purchase price” at expiration of: 46(1,750)(250) = $20,125,000

At 1% interest, the amount to invest in T-bills today is $20,125,000 / 1.01^{3/12} = $20,075,000. This is somewhat higher than the desired $20,000,000 because the number of contracts purchased was rounded up.
Manager B has a large position in U.K. stocks that are similar to a major U.K. stock index. She wishes to create GBP 15,000,000 of synthetic cash earning 2.0% for a six-month period. The futures index contract is priced at 3,700 with a multiplier of 10. The stocks have a dividend yield of 3.0%.

**Calculate** the number of contracts to buy or sell and the amount of synthetic cash created.

**Answer:**

Sell 409 contracts. (Because no betas were given, it is presumed the portfolio beta is the same as the futures beta. Purely for illustration, assume they are both 0.95. They will have no affect.)

\[
N_f = \left[\frac{(0.0 - 0.95)}{0.95}\right] \times \left[\frac{(15,000,000 \times 1.02^{6/12})}{(3,700 \times 10)}\right] \\
= (-1) \times (15,149,257 / 37,000) = -409.44 \approx -409
\]

This is a full “price” at expiration of: 409(3,700)(10) = 15,133,000

At 2% interest, the present value invested today in risk-free assets is 15,133,000 / 1.02^{6/12} = GBP 14,983,903. This is somewhat less than the desired GBP 15,000,000 because the amount hedged (number of contracts sold) was rounded down.

**Why Future Value (FV) Is Used in the Synthetic Calculations**

The use of the risk-free rate and FV would, in a perfect hedge, mean the synthetic position completely replicates the beginning and ending results that would have been obtained if the desired synthetic position had been actually held. This can be seen by evaluating the results achieved with the rounded number of contracts as either a theoretical delivery of the underlying (which may or may not be allowed by the contract) or by comparing the initial investment as well as ending gains or losses in the synthetic position with those from having held an actual position.

- For delivery analysis, it is expedient and acceptable to view the contract price as a price per share and the number of contracts \( \times \) contract multiplier as the number of shares.
- Recall the contract price is based on the cash and carry arbitrage relationship studied at Level II. \( F_0 \) is the FV of \( S_0 \) minus the FV of any dividends to occur during contract life on the underlying.
- The dividend yield is, therefore, already “priced” into the contract price. Alternatively, the dividend yield would have been earned if the underlying were owned.
- The analysis must be based on the rounded number of contracts actually used because fractional contracts do not exist.
Performance of a Synthetic Position if Delivery is Allowed

To illustrate, return to the example of Manager A who wishes to create $20,000,000 of synthetic S&P 500 stock exposure for three months when the risk-free rate is 1%. The S&P contract is priced at 1,750, the dollar multiplier is 250, the underlying index has a dividend yield of 2.5%, and it is priced at 1,756.461648 (6 decimals are used only to demonstrate the accuracy of the analysis).

The manager purchased 46 contracts, not the desired fractional number of 45.82.

This is conceptually equivalent to buying 46 × 250 = 11,500 shares at a forward price of 1,750.

This is a PV amount invested today of:

\[
\frac{46(1,750)(250)}{1.01^{3/12}} = \frac{20,125,000}{1.01^{3/12}} = 20,075,000
\]

That, of course, also means that if $20,075,000 were invested today at the risk-free rate of 1%, it will be worth $20,125,000 and it will have earned interest of $50,000. The ending amount can be used to pay the contracted price of 1,750 on 11,500 shares. The investor will then own 11,500 shares worth $S_T$.

Alternatively, the $20,075,000 could have been used to buy shares initially. If they were purchased, the dividends can then be reinvested in the purchase of more shares.

- The shares purchased today will be the contracted number of shares discounted by the dividend yield. This is (46 × 250) / 1.025^{3/12} = 11,429.227
- A more direct way to calculate this is the initial investment amount divided by today's share price. This is: $20,075,000 / 1,756.461648 = 11,429.228
- The two approaches are equivalent because the spot and future price relationship reflects the initial dividend yield and risk-free rate. (Ignoring the small rounding discrepancy.)
- Reinvesting the dividends, this will be 11,429.227 × 1.025^{3/12} = 11,500 shares worth $S_T$ at the end of the contract period.

The synthetic and actual ownership had the same initial investment, and both result in owning 11,500 shares worth $S_T$ at contract expiration.
Performance of a Synthetic Position Based on G/L

Professor’s Note: The analysis of equivalence of synthetic and actual positions based on G/L or examples where both spot and futures price are given is not covered directly in the CFA text. It is included because we get questions from candidates about why the CFA text presumes delivery of the underlying items to settle the contract when this is generally not allowed. You may skip this entire note if you wish.

Assuming, as is done in the CFA text, that dividend yield and risk-free rates are compounded annual rates, then \( F_0 = S_0 \left[ (1 + r_f) / (1 + \text{dividend yield}) \right]^T \). In the example, this is \( 1,750 = 1,756.461648(1.01/1.025)^{0.25} \).

For illustration, assume the ending stock and contract price are 1,900. They will be equal based on convergence.

The initial 11,429.227 shares at 1,756.461648 are worth $20,075,000. (Precision requires using infinite decimal places in all calculations). The ending shares (with dividend reinvestment) of 11,500 shares at 1,900 are worth $21,850,000. This is a gain of $1,775,000.

Recall the synthetic position holds cash equivalents and earns interest of $50,000. The futures price increases from the initial purchase price of 1,750 to 1,900 for a gain of \((1,900 – 1,750) \times 46 \times 250 = 1,725,000, \text{ making the total gain } $1,775,000.\)

Based on either delivery or G/L analysis, the synthetic and actual ownership produce the same result.
LOS 26.c: Explain the use of stock index futures to convert a long stock position into synthetic cash.

Example: Synthetic Cash Position

Manager C holds equity positions similar to the Russell 2000 and wishes to synthetically convert $50,000,000 to cash equivalents for five months. He decides to use a contract overlay position rather than sell the stocks and then have to repurchase them. The Russell 2000 futures contract price is 1,135 with a multiplier of 500. The Russell Index dividend yield is 1.7%, and the zero coupon bond rate is 0.9%.

Calculate: i) the number of contracts for the position, ii) the effective beginning investment in cash equivalents, iii) the effective number of shares in the index converted to cash, and iv) assuming the index closes at 1,057, demonstrate the strategy is equivalent to having invested at the risk-free rate.

Answers:

i) The betas of the index and portfolio were not given, are assumed to be equal, and, therefore, do not affect the calculation. The number of contracts to sell is:

$$\frac{50,000,000(1.009^{5/12})}{(1,135)(500)} = \frac{50,187,010}{567,500} = 88.44 \approx 88$$

ii) The effective initial amount of cash equivalents is:

$$\frac{[88(500)(1,135)]}{1.009^{5/12}} = \frac{49,940,000}{1.0037402} = 49,753,910$$

iii) The effective number of shares converted to cash is:

$$88(500) / 1.017^{5/12} = 44,000 / 1.00370485 = 43,692.04$$

With dividends reinvested, this is ending shares of:

$$43,692.04(1.017^{5/12}) = 44,000$$

iv) At contract expiration, the index and contract price will converge to 1,057. The pay off on the short contract position is a gain because the contract price declined. The gain is:

$$(1,135 - 1,057)(88)(500) = 3,432,000$$

The ending value of the shares is:

$$44,000(1,057) = 46,508,000$$
This makes the total ending value $49,940,000 versus an initial synthetic cash position of $49,753,910 for an effective annual return:

\[
\frac{49,940,000}{49,753,910}^{\frac{1}{12}} - 1 = 0.9\%
\]

This synthetic position produced a return equivalent to the initial risk-free rate of 0.9%.

**Altering Bond Exposure Using Contracts**

**LOS 26.d:** Demonstrate the use of equity and bond futures to adjust the allocation of a portfolio between equity and debt.

The same formula used to adjust equity beta can be used to adjust bond duration by using duration instead of beta in the calculation.

**Target Duration**

The number of futures contracts needed to combine with a bond to achieve a targeted portfolio duration is:

\[
\text{number of contracts} = (\text{yield beta}) \left( \frac{\text{MD}_T - \text{MD}_P}{\text{MD}_F} \right) \left( \frac{V_p}{P_f \text{ (multiplier)}} \right)
\]

where:
- \(V_p\) = current value of the portfolio
- \(P_f\) = futures price
- \(\text{MD}_T\) = target (desired) modified duration
- \(\text{MD}_P\) = modified duration of the portfolio
- \(\text{MD}_F\) = modified duration of the futures
Cross-Reference to CFA Institute Assigned Reading #26 – Risk Management Applications of Forward and Futures Strategies

Professor’s Note: Superficially the formula looks different than one seen in fixed income. The results are the same.

\[
\text{number of contracts} = \left( \frac{D_T - D_F}{D_{CTD}^P_{CTD}} \right) P^*_F (CTD conversion factor)
\]

- If yield beta is not given, it is implicitly assumed to be 1.0 and irrelevant. If it is given, include it as a multiplier.
- One formula uses \( D \), and one uses \( MD \). This is just notation difference in the two readings. The duration of the CTD is the duration of the contract.
- \( P^*_F \) (multiplier) is the full value of the contract. The fixed income reading assignment gave that number directly and used it. This assignment shows you it is calculated as \( P^*_F \) (multiplier).
- This reading assignment uses the price of the futures contract while the fixed income assignment used price of the CTD and its conversion factor. The conversion factor is the link between these two prices, making the two formulas identical mathematically.

The bottom line is to know both formulas and use the one for which inputs are given.

**Example: Altering duration**

A) The manager has a bond portfolio with a value of $103,630 and a holding period of one year. The 1-year total futures price is $102,510. The modified duration of the portfolio and futures contracts are 1.793 and 1.62, respectively. The yield beta is 1.2.

Calculate the number of contracts to reduce the portfolio duration to 0.

Answer:

\[
\text{number of contracts} = (1.2) \left( \frac{0 - 1.793}{1.62} \right) \left( \frac{\$103,630}{\$102,510} \right) = -1.34
\]

Sell one contract at 102,510. This is going to produce a rather significant rounding error.

B) Suppose the manager wants to change the portfolio duration from 1.793 to 3.0.

Calculate the number of contracts to increase duration to 3.

Answer:

\[
\text{number of contracts} = (1.2) \left( \frac{3 - 1.793}{1.62} \right) \left( \frac{\$103,630}{\$102,510} \right) = 0.9 \rightarrow \text{buy one contract at 102,510}
\]
Adjusting Portfolio Asset Allocation

Adjusting asset allocation uses the same number of contracts formulas but requires multiple steps:

• Adjustments are often stated as percent allocations; however, the calculations require dollar or other nominal amounts (e.g., a 10% shift of a EUR 50M portfolio is a EUR 5M $V_p$).
• Changing an allocation requires selling contracts to remove one exposure and buying contracts to create a different exposure.

Example: Altering debt and equity allocations

A manager has a $50 million portfolio that consists of 50% stock and 50% bonds (i.e., $25 million each).

• The beta of the stock position is 0.8.
• The modified duration of the bond position is 6.8.

The manager wishes to achieve an effective mix of 60% stock (i.e., $30 million) and 40% bonds (i.e., $20 million). Because the move is only temporary, and rather than having to decide which bonds to sell and which stocks to buy to achieve the desired mix, the manager will use futures contracts.

• The price of the stock index futures contract is $300,000 (including the multiplier), and its beta is 1.1.
• The price, modified duration, and yield beta of the futures contracts are $102,000, 8.1, and 1, respectively.

Determine the appropriate strategy.

Answer:

The desired shift is $5,000,000. Sell bond contracts to reduce duration to 0 on a $5,000,000 position:

\[
\text{number of bond futures} = (\text{yield beta}) \left( \frac{\text{MD}_{T} - \text{MD}_{P}}{\text{MD}_{f}} \right) \left( \frac{V_p}{P_f \text{(multiplier)}} \right)
\]

\[
= (1) \left( \frac{0.0 - 6.8}{8.1} \right) \left( \frac{$5,000,000}{$102,000} \right) = -41.2
\]

Sell 41 bond contracts at $102,000.
Buy equity contracts targeting the desired beta of 0.8 on $5,000,000:

\[
\text{number of equity index futures} = \left( \frac{\beta_t}{\beta_f} \right) \left( \frac{V_p}{P_f \text{ (multiplier)}} \right)
\]

\[
= \left( \frac{0.8}{1.1} \right) \left( \frac{$5,000,000}{$300,000} \right)
\]

\[
= (0.727) \times (16.666) = 12.12
\]

Buy 12 equity contracts at 300,000.

**Adjusting the Equity Allocation**

LOS 26.e: Demonstrate the use of futures to adjust the allocation of a portfolio across equity sectors and to gain exposure to an asset class in advance of actually committing funds to the asset class.

The same process can be used to make any portfolio asset allocation, as long as the appropriate contracts to buy and sell are available.

**Example: Changing equity allocations**

A manager of $20 million of mid-cap equities would like to move half of the position to small-cap equities. The beta of the mid-cap position is 1.1, and the average beta of small-cap stocks is 1.5. The betas of the corresponding mid- and small-cap futures contracts are 1.05 and 1.4, respectively. The mid- and small-cap futures total prices are $244,560 and $210,500, respectively. **Determine** the appropriate strategy.

**Answer:**

The desired reallocation is $10,000,000. Sell mid-cap contracts and buy small-cap contracts.

**number of contracts**_{mid\_cap} = \left( \frac{0 - 1.1}{1.05} \right) \left( \frac{$10,000,000}{$244,560} \right) = -42.84

Sell 43 mid-cap contracts at 244,560.

**number of contracts**_{small\_cap} = \left( \frac{1.5 - 0}{1.4} \right) \left( \frac{$10,000,000}{$210,500} \right) = 50.90

Buy 51 small-cap contracts at 210,500.
Preinvesting refers to buying contracts in anticipation of cash that will be received. Buying contracts does not require initial cash flow, which makes contracts a natural vehicle for such transactions. It is assumed the account has other assets that can be posted to meet margin requirements. Because this is hedging a future value amount, it is most appropriate to refer to this as a synthetic position.

Example: Preinvesting

A portfolio manager knows that $5 million in cash will be received in a month. The portfolio under management is 70% invested in stock with an average beta of 0.9 and 30% invested in bonds with a duration of 4.8. The most appropriate stock index futures contract has a total price of $244,560 and a beta of 1.05. The most appropriate bond index futures have a yield beta of 1.00, an effective duration of 6.4, and a total price of $99,000. Determine the appropriate strategy to synthetically preinvest the $5 million in the same proportions as the current portfolio.

Answer:

The goal is to create a $3.5 million equity position (0.7 × $5 million) with a beta of 0.9 and a $1.5 million bond position (0.3 × $5 million) with a duration of 4.8:

number of stock futures =

\[
\frac{0.9 - 0}{1.05} \cdot \frac{3,500,000}{244,560} = 12.27, \text{ buy 12 contracts at 244,560}
\]

number of bond futures =

\[
\frac{4.8 - 0}{6.4} \cdot \frac{1,500,000}{99,000} = 11.36, \text{ buy 11 contracts at 99,000}
\]

The manager should take a long position in 12 stock index futures and 11 bond index futures.

Professor’s Note: The anticipated $5,000,000 has no duration or beta.

Exchange Rate Risk

LOS 26.f: Explain exchange rate risk and demonstrate the use of forward contracts to reduce the risk associated with a future receipt or payment in a foreign currency.
Cross-Reference to CFA Institute Assigned Reading #26 – Risk Management Applications of Forward and Futures Strategies

Professor’s Note: You are responsible for any of the calculations in this section, but they are covered elsewhere. It is important you focus on the implications of the math discussed here, not just the plug and do. We are returning to the issue of investing in foreign assets carrying two sources of return and risk. Remember that perfect hedging of the currency risk is unlikely, as that would require knowing the ending value of the asset at the start of the hedging period.

Three types of foreign exchange risk are:

1. **Transaction exposure** occurs when a cash flow in a foreign currency will be received or paid at a future date. This can be hedged. If it will be received, sell it forward. If it will be paid, buy it forward.

2. **Economic exposure** is just as real as transaction exposure but less visible initially. It refers to situations when changes in currency value affect business competitiveness and it can have many causes. For example, consider a company that:
   - Exports and sells its products in foreign markets. If the domestic currency appreciates, its products become less competitive internationally, leading to a drop in revenue and, likely, in profits and stock price.
   - Purchases and imports from foreign markets items used in the business. If the domestic currency depreciates, costs in domestic currency units increase, leading to lower profits and, likely, in stock price.
   - Only operates domestically with no imports or exports but changes in currency value affects its competitors (or suppliers or customers). Any of these could affect the company’s business, profits, and stock price.

   Economic exposure is a real risk to cash flow and might be desirable to hedge, but it could also be difficult to quantify and be ongoing. (Hint: this is very similar to the issue of correlation between LMR and LCR and the minimum variance hedge ratio discussed in other reading assignments).

3. **Translation exposure** is the risk of converting foreign denominated financial statements in to domestic currency units. (Hint: you might vaguely remember the two methods taught at Level 2, all current and temporal method. Recall they produce very different results and numbers for the gain or loss. You will not find any examples of hedging translation exposure because it is generally not seen as a real cash flow risk. Of course if a question said here is a number, hedge it, you would do so on the exam).

Derivatives are used most often to hedge exposure with transaction exposure the one most commonly hedged:

- Being **long the currency** (in this context) means you have contracted to receive the foreign currency. The chief concern is that the foreign currency might depreciate before it is received. For example, assume a U.S. exporter has contracted to sell $10 million in merchandise for €9 million to a European trade partner, which reflects the current €0.90/$ exchange rate.
The payment will be made in 60 days, and in that time, the dollar may appreciate to €0.91/$. In that case, the dollar revenue at that time will be \((€9,000,000)(€/0.91) = $9,890,110\), which is a loss of over $100,000.

- **Being short the currency** means you have contracted to pay the foreign currency, and the concern is that the currency will appreciate. For example, a U.S. importer contracted to purchase €10 million of a foreign good when the exchange rate was €0.9/$, which translated to a cost of $11,111,111. If the dollar depreciates to €0.89/$ before the payment is made, this would increase the dollar cost to $11,235,955.

In our previous discussion, we showed that transaction exposure can be either long or short. Figure 1 will help you remember the strategy a manager should take to hedge these exposures:

Figure 1: Strategies for Hedging Expected Currency Positions

<table>
<thead>
<tr>
<th>Contractual Agreement</th>
<th>Position</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiving foreign currency</td>
<td>Long</td>
<td>Sell forward contract</td>
</tr>
<tr>
<td>Paying foreign currency</td>
<td>Short</td>
<td>Buy forward contract</td>
</tr>
</tbody>
</table>

**Example: Managing exchange rate risk**

Mach, Inc., is a U.S.-based maker of large industrial machines and has just received an order for some of its products. The agreed-upon price is £5 million (British pounds), and the delivery date is 60 days. The current exchange rate is $1.42 per pound, and the 60-day forward rate is $1.43 per pound. **Explain** the best way for Mach, Inc., to hedge the corresponding exchange rate risk.

**Answer:**

On the day the order comes in, Mach, Inc., effectively has a long position in pounds; therefore, it should take a short position in a forward contract. This contract would obligate Mach, Inc., to deliver the pounds that it will receive for dollars. Ideally, the contract would be to exchange the £5,000,000 for:

\[7,150,000 = (5,000,000)(1.43)\]

According to the contract, in 60 days, Mach will exchange the £5,000,000 for $7,150,000. If it does not hedge and the realized spot rate in 60 days is $1.429, Mach will receive only $7,145,000 = 5,000,000($1.429), or $5,000 less than with the hedged position.
Example: Exchange rate risk

U.S.-based Goblet, Inc., imports wine from France. It has just contracted to pay €8 million for a shipment of wine in 30 days. The current spot rate is €0.8/$, and the 30-day forward rate is €0.799/$. Explain the strategy Goblet, Inc., could employ to eliminate the exchange rate risk.

Answer:

Because Goblet, Inc., will have to pay euros, it is short the currency and should go long (buy) the forward contract. Goblet, Inc., should enter a forward contract that will allow it to buy the €8,000,000 it will need for the contract for $10,012,516 = €8,000,000 ($/€0.799).

What if Goblet, Inc., had not hedged, and the exchange rate is €0.7995/$ in 30 days? If this is the case, Goblet, Inc., would get the necessary €8 million from converting $10 million at that spot rate. The dollar cost would be $10,006,254 = €8,000,000 ($/€0.7995). Thus, without the contract, Goblet, Inc., would have been $6,262 dollars better off in the spot market.

Discussion

We see that in one case, the firm benefited from the hedged position, and in the other case, the firm was hurt (i.e., suffered an opportunity cost of $6,262). So why shouldn't firms just take the positives and negatives and let them cancel out over time? Why not try to predict exchange rates and only hedge in those cases where the prediction says that the rates will turn against you? Some firms do this but predicting currency change is risky and takes time and resources. Other firms systematically hedge (or don't hedge) and just focus on the underlying business of the firm.

For the Exam: Currency risk management is earning greater and greater attention in the Level III curriculum as the curriculum turns more and more to a global focus. Be ready to deal with exchange rate risk in both sessions of the exam, as part of an item set in the afternoon or part of a case (i.e., constructed response essay) in the morning.

Hedging Limitations

LOS 26.g: Explain the limitations to hedging the exchange rate risk of a foreign market portfolio and discuss feasible strategies for managing such risk.

An equity investment in a foreign market has both equity risk and foreign exchange risk. That is, the foreign position will increase or decrease in value according to the activity in
the foreign market, and then the domestic investor will face additional return volatility because of the uncertainty caused by fluctuations of the exchange rate. A foreign equity position may increase by 10%, but if the foreign currency depreciates by that much, the net change to the domestic investor is approximately zero.

The two hedging strategies utilized by global portfolio managers to manage the risk of a foreign-denominated portfolio involve selling forward contracts on the foreign market index (to manage market risk) and selling forward contracts on the foreign currency (to manage the currency risk). They can choose to hedge one or the other, both, or neither. Their four choices can be summed up as follows:

1. Hedge the foreign market risk and accept the foreign currency risk.
2. Hedge the foreign currency risk and accept the foreign market risk.
3. Hedge both risks.
4. Hedge neither risk.

**Hedging Market Risk**

To hedge the market value (i.e., market risk) of a foreign investment, the manager can short (i.e., sell forward) the foreign market index. The degree to which the portfolio is correlated with the market index will determine the effectiveness of the hedge. If the manager shorts the appropriate amount of the index and it is perfectly correlated with the portfolio of investments, the return from the hedging strategy must be the foreign risk-free rate.

If the same manager then chooses to hedge the currency risk, she knows the exact value of the foreign currency to hedge, and the return to the (double) hedging strategy must be the manager’s domestic risk-free rate.

**Hedging Currency Risk**

An obvious problem faced when trying to hedge the foreign currency risk of a foreign investment is its uncertain future value. Managers use various strategies for managing the currency risk of a foreign portfolio, including:

- Hedging a minimum future value below which they feel the portfolio will not fall.
- Hedging the estimated future value of the portfolio.
- Hedging the initial value (i.e., the principal).

None of these strategies can eliminate all the currency risk. For example, even if management has determined a minimum future value below which the portfolio will not fall, they are still exposed to values above that. If they hedge the principal, portfolio gains are unhedged. A loss in portfolio value would represent an over-hedge (i.e., management has agreed to deliver too much of the foreign currency).
Another proposed strategy is doing nothing (i.e., hedging choice #4 noted previously). As long as the market and currency risks are not highly correlated, changes in the two values will tend to offset one another.

Hedging with Futures or Forwards?

Calculating and constructing the hedge treats futures and forwards interchangeably. There could be occasions when one or the other is favored for practical reasons. Some differences to know are:

• Futures are standardized contracts while forwards can be customized as to amount and expiration date.
• Futures are obligations of the exchange clearinghouse while forwards have counterparty risk.
• Futures are more regulated and transparent, and they require margin.

Empirically:

• Most bond and equity hedging is done with futures even though this usually creates some cross-hedge or basis risk because the futures provide ongoing liquidity and are continually priced.
• Hedging of interest payments or receipts is usually done with forwards (FRAs), so exact amounts and dates can be hedged.
• Likewise, currency hedging generally uses forwards to tailor amounts and dates.
• Eurodollar futures are a very large market but are mostly used by dealers and market makers to hedge their own business needs and positions and not used directly by final customers.
LOS 26.a
Buy contracts to increase beta or duration. Sell contracts to decrease beta or duration.

\[
\text{number of contracts} = \left( \frac{\beta_T - \beta_P}{\beta_f} \right) \left( \frac{V_p}{P_f \text{ (multiplier)}} \right)
\]

where:
- \(\beta_T\) = desired portfolio beta
- \(\beta_P\) = portfolio beta
- \(\beta_f\) = equity futures contract beta
- \(V_p\) = current value of the portfolio
- \(P_f\) = futures price

The same formula is used to calculate the number of bond contracts by replacing beta with duration. Yield beta is included as a multiplier if it is given, otherwise it is assumed to be 1.0.

LOS 26.b and c
The same formulas used to adjust beta and duration can also be used to create synthetic positions. \(V_p\) must be replaced with a future value amount \(V_p \times (1 + r_f)^T\).

Often the desired change in beta or duration is equal to the beta or duration of the contract being used (e.g., \(-1.1/1.1\) or \(0.97/0.97\)). This produces an absolute ratio of 1.00 and, as a result, the betas and durations do not affect the number of contracts calculation.

A synthetic equity position requires buying contracts and holding sufficient cash earning \(r_f\) to pay for the contracts at contract expiration.

The initial required cash position is:

\[
\frac{(\text{number of contracts}_\text{rounded})(P_f)(\text{multiplier})}{(1 + R_f)^T}
\]

A synthetic cash position requires selling contracts and holding sufficient shares (with dividends reinvested in more shares) to provide the shares to deliver and close the short position.

The initial number of shares required is:

\[
\frac{(\text{number of contracts}_\text{rounded})(\text{multiplier})}{(1 + \text{dividend yield})^T}
\]
LOS 26.d and e
Adjusting asset allocation uses the same number of contracts formulas but requires multiple steps:

- Adjustments are often stated as percent allocations; however, the calculations require dollar or other nominal amounts (e.g., a 10% shift of a EUR 50M portfolio is a EUR 5M Vp).
- Changing an allocation requires selling contracts to remove one exposure and buying contracts to create a different exposure.

LOS 26.f
1. Economic exposure is the risk that changes in currency value may affect competitive position, sales, and profits.

2. Translation exposure is the risk of converting financial statements in one currency to another currency.

3. Transaction exposure is the risk that changes in exchange rates will directly affect the value of a contracted payment in or receipt of a foreign currency.

Derivatives are often used to hedge transaction exposure:

**Strategies for Hedging Expected Currency Positions**

<table>
<thead>
<tr>
<th>Contractual Agreement</th>
<th>Position</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiving foreign currency</td>
<td>Long</td>
<td>Sell forward contract</td>
</tr>
<tr>
<td>Paying foreign currency</td>
<td>Short</td>
<td>Buy forward contract</td>
</tr>
</tbody>
</table>

LOS 26.g
An equity investment in a foreign market has both equity risk and foreign exchange risk. The investment is exposed to both the change in value of the foreign investment measured in the foreign currency and the change in value of the foreign currency. This leads to four possible hedging strategies: hedge neither risk, hedge one but not the other risk, or hedge both risks. If both risks are perfectly hedged, all risk is removed and the hedged results should equal the investor’s (not the foreign asset’s) risk-free rate.
CONCEPT CHECKERS

1. The duration of a bond portfolio is 6, and the duration of the most appropriate bond futures contract is 4. The size of the portfolio is 24 times the total futures price. The yield beta of the futures contract is 1. The most appropriate strategy to completely hedge the portfolio against changes in interest rates is:
   A. short 1 futures contract.
   B. go long 24 futures contracts.
   C. short 36 contracts.

2. A portfolio manager expects a large cash inflow in the near future and wishes to preinvest the cash flow to earn an equity market return. The most appropriate strategy is to:
   A. take a short position in a stock index futures contract today.
   B. take a long position in a stock index futures contract today.
   C. take a short position in a stock index futures contract when the cash is received.

3. A domestic firm experiences a loss of revenue from the loss in sales caused by changes in value of the domestic currency. This type of loss is referred to as:
   A. translation risk.
   B. transaction risk.
   C. economic risk.

4. Sweat Pants, Inc., a U.S.-based firm, has entered into a contract to import £2,000,000 worth of wool from a firm in Scotland, and the spot exchange rate is $1.50/£. Management of Sweat Pants wants to alleviate the risk associated with the foreign currency. The forward exchange rate corresponding with the delivery of the wool is $1.455/£. Which of the following would probably be the best tactic to use to alleviate the foreign exchange risk for Sweat Pants?
   A. Sell £2,000,000 forward and agree to receive $2,910,000.
   B. Buy £2,000,000 forward and agree to deliver $2,910,000.
   C. Sell £2,000,000 forward and agree to receive $1,374,570.

5. A French investor has invested in a large, diversified portfolio of Japanese stocks. Which of the following tactics could be used to hedge the investment and target a return equal to the French risk-free rate?
   A. Buy euros forward and sell the foreign equity index.
   B. Sell euros forward and sell the foreign equity index.
   C. Buy euros forward and buy the foreign equity index.
6. Portfolio Management, Inc. (PMI) expects a cash flow of $10,000,000 in two months. The composition of the PMI portfolio is 40% large-cap equities, 40% small-cap equities, and 20% bonds. Using the following information, determine the appropriate strategy for PMI managers to synthetically preinvest the $10,000,000, so that it earns returns equivalent to those of their current positions.

- Large-cap beta = 0.9; small-cap beta = 1.35; bond duration = 6.3; yield beta = 1.0.
- Large-cap futures beta = 1.0; small-cap futures beta = 1.30.
- Treasury futures duration = 5.8.
- Large-cap futures price = $1,400, multiplier = $250; (= $350,000).
- Small-cap futures price = $1,100, multiplier = $250; (= $275,000).
- Treasury futures price = $100,000.

7. A manager has a position in Treasury bills worth $100 million with a yield of 2%. For the next three months, the manager wishes to have a synthetic equity position approximately equal to this value. The manager chooses S&P 500 Index futures, and that index has a dividend yield of 1%. The futures price is $1,050, and the multiplier is $250. Determine how many contracts this will require and the initial value of the synthetic stock position.

8. A manager of a $10,000,000 portfolio wants to decrease the beta from the current value of 1.6 to 1.2. The beta on the futures contract is 1.25, and the total futures price is $250,000. Using the futures contracts, calculate the appropriate strategy.
9. A manager has a $100 million portfolio that consists of 70% stock and 30% bonds. The manager wishes to achieve an effective mix of 50% stock and 50% bonds.
   • The beta of the stock position is 1.2.
   • The modified duration of the bond position is 4.0.
   • The price and beta of the stock index futures contracts are $225,000 and 1.0, respectively.
   • The price, modified duration, and yield beta of the futures contracts are $100,500; 5; and 1, respectively.

Determine the appropriate strategy.

10. A manager of $10 million of large-cap equities would like to shift 25% of the position to mid-cap equities. The beta of the large-cap position is 0.8, and the average beta of mid-cap stocks is 1.2. The betas of the corresponding large and mid-cap futures contracts are 0.75 and 1.25, respectively. The large- and mid-cap total futures prices are $9,800 and $240,000, respectively. Determine the appropriate strategy.

For more questions related to this topic review, log in to your Schweser online account and launch SchweserPro™ QBank; and for video instruction covering each LOS in this topic review, log in to your Schweser online account and launch the OnDemand video lectures, if you have purchased these products.
ANSWERS – CONCEPT CHECKERS

1. C  number of contracts = \( (\text{yield beta}) \left[ \frac{-\text{MD}_P}{\text{MD}_f} \right] \left[ \frac{V_P}{P_f (\text{multiplier})} \right] \) = (1) \( \left[ \frac{-6}{4} \right] \) (24) = -36

   Short 36 contracts to hedge the portfolio.

2. B  The number of equity index futures is determined by dividing the expected cash position by the total price of the equity index.

3. C  This is economic exchange rate risk.

4. B  To alleviate the risk associated with moving foreign exchange values, Sweat Pants will enter a forward contract in which they agree to deliver $2,910,000 = ($1.455/£) (£2,000,000) and receive the £2,000,000 needed to pay for the wool.

5. A  The French investor is exposed to two sources of risk: change in value of the foreign stock market and change in value of the foreign currency. If both risks are eliminated, the position is risk-free and the investor earns his own domestic (French) risk-free rate.

   To hedge the foreign market, sell Japanese stock index futures forward. To hedge the foreign currency, sell the JPY forward (which is buy the investor’s domestic currency, EUR) forward.

6.  Because the portfolio is currently 40/40/20 large cap, small cap, and bonds, management should assume long positions in futures contracts in those proportions:

   40% small cap and large cap = $4,000,000 each; 20% bonds = $2,000,000

   Management should buy large-cap equity futures, small-cap equity futures, and Treasury futures:

   \[
   \text{# equity futures} = \frac{\beta_T - \beta_F}{\beta_F} \left[ \frac{V_P}{P_f (\text{multiplier})} \right]
   \]

   \[
   \text{# large-cap contracts} = \frac{0.9 - 0}{1.0} \left[ \frac{4,000,000}{350,000} \right] = 10.29, \text{ buy 10 at 350,000}
   \]

   \[
   \text{# small-cap contracts} = \frac{1.35 - 0}{1.3} \left[ \frac{4,000,000}{275,000} \right] = 15.10, \text{ buy 15 at 275,000}
   \]

   \[
   \text{# Treasury futures} = \beta_{\text{Yield}} \left[ \frac{D_T - D_F}{D_F} \right] \left[ \frac{V_P}{P_f} \right]
   \]

   \[
   \text{# contracts} = (1) \left[ \frac{6.3 - 0}{5.8} \right] \left[ \frac{2,000,000}{100,000} \right] = 21.72, \text{ buy 22 at 100,000}
   \]

7.  number of contracts = \( \frac{($100,000,000)(1.02)^{0.25}}{(1,050)(250)} \) = 382.84, buy 383 at 1,050

   This is equivalent to an initial investment of \( [(383)(1,050)(250)] / (1.02)^{0.25} = $100,041,003. \)
8. The number of contracts is 
\[
\frac{(1.2 - 1.6)(-10,000,000)}{(1.25)(-250,000)} = -12.8
\]
sell 13 contracts at 250,000

9. The desired shift in allocation is 20% of $100M = $20M. Sell stock contracts and buy bond contracts to achieve the desired 0 beta and 4.0 duration.

   number of stock futures = \[
   \frac{(0 - 1.2)(-20,000,000)}{(1.0)(-225,000)} = -106.67
\]
sell 107 at 225,000

   number of bond futures = \[
   \frac{(4 - 0.0)(-20,000,000)}{(5)(-100,500)} = +159.20
\]
buy 159 at 100,500

10. The desired allocation shift is 25% of $10M = $2.5M. Sell large-cap and buy mid-cap equity contracts to achieve a large-cap beta of 0 and mid-cap beta of 1.2.

   number of contracts = \[
   \frac{(0 - 0.8)(-2,500,000)}{(0.75)(-9,800)} = -272.11
\]
sell 272 at 9,800

   number of contracts = \[
   \frac{(1.2 - 0)(-2,500,000)}{(1.25)(-240,000)} = 10.0
\]
buy 10 mid-cap at 240,000
The following is a review of the Risk Management Applications of Derivatives principles designed to address the learning outcome statements set forth by CFA Institute. Cross-Reference to CFA Institute Assigned Reading #27.

**RISK MANAGEMENT APPLICATIONS OF OPTION STRATEGIES**

**Exam Focus**

As you read through this topic review, you will notice that almost all the LOS are quantitative, but do not just focus on the calculations. Instead, learn the underlying concepts. For example, you might memorize the equations associated with a bull spread as presented in the CFA text and not realize end of chapter questions presume you would have noticed the same result can be achieved in other ways. Interest rate collars have been a favorite on the exam and many candidates are more familiar with options based on price, rather than options based directly on interest rates. Be prepared to work with either. For the exam, be sure you know the construction and payoffs for the strategies as well as their similarities. Be prepared for both item set and constructed response questions and for questions that integrate this material into determining the best solution for a given investor for a given set of facts.

**Warm-Up: Basics of Put Options and Call Options**

*Professor’s Note: The next several pages up to Covered Calls and Protective Puts are review and have only one purpose: you must know the four basic payoff patterns of long or short a call or put and how to compute intrinsic value. These items are the starting point of the material assigned at Level III. Be sure you know it or nothing that follows makes much sense.*

Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

- $X =$ strike price or exercise price specified in the option contract (a fixed value)
- $S_t =$ price of the underlying asset at time $t$
- $C_t =$ market value of a call option at time $t$
- $P_t =$ market value of a put option at time $t$
- $t =$ time subscript, which can take any value between 0 and $T$, where $T$ is the maturity or expiration date of the option
### Call Options

A call option gives the owner the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the buyer or the holder of the long position. The buyer benefits, at the expense of the option seller, if the underlying stock price is greater than the exercise price. The option seller is also called the writer or holder of the short position.

At maturity, time $T$, if the price of the underlying stock is less than or equal to the strike price of a call option (i.e., $S_T \leq X$), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than the exercise price (i.e., $S_T > X$) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ($S_T - X$). The “payoff” (at the option’s maturity) to the call option seller, which will be at most zero, is the mirror image (opposite sign) of the payoff to the buyer.

Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 1 illustrates the payoff of a call with an exercise price equal to 50.

**Professor's Note:** A payoff graph ignores the initial cost of the option.

---

**Figure 1: Payoff of Call With Exercise Price Equal to 50**

![Payoff Graph](image-url)
Example: Payoff to the writer of a call option

An investor writes an at-the-money call option on a stock with an exercise price of $50 (X = $50). If the stock price rises to $60, what will be the payoff to the owner and seller of the call option?

Answer:

The call option may be exercised with the holder of the long position buying the stock from the writer at $50 for a $10 gain. The payoff to the option buyer is $10, and the payoff to the option writer is negative $10. This is illustrated in Figure 1, and as mentioned, does not include the premium paid for the option.

This example shows just how easy it is to determine option payoffs. At expiration time T (the option's maturity), the payoff to the option owner, represented by \( C_T \), is:

\[
C_T = \begin{cases} 
S_T - X & \text{if } S_T > X \\
0 & \text{if } S_T \leq X 
\end{cases}
\]

Discussion

Another popular way of writing this is with the “\( \max(0, \text{variable}) \)” notation. If the variable in this expression is greater than zero, then \( \max(0, \text{variable}) = \text{variable} \); if the variable’s value is less than zero, then \( \max(0, \text{variable}) = 0 \). Thus, letting the variable be the quantity \( S_0 - X \), we can write:

\[ C_T = \max(0, S_T - X) \]

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner unless otherwise stated. We should note that \( \max(0, S_t - X) \), where \( 0 < t < T \), is also the payoff if the owner decides to exercise the call option early. In this topic review, we will only consider time \( T \) in our analysis. Determining how to compute \( C_t \) when \( 0 < t < T \) is a complex task to be addressed later in this topic review.

Although our focus here is not to calculate \( C_t \), we should clearly define it as the initial cost of the call when the investor purchases at time 0, which is \( T \) units of time before \( T \). \( C_0 \) is the call premium. Thus, we can write that the profit to the owner at \( t = T \) is:

\[ \text{profit} = C_T - C_0 \]

This says that at time \( T \), the owner’s profit is the option payoff minus the premium paid at time 0. Incorporating \( C_0 \) into Figure 1 gives us the profit diagram for a call at expiration, and this is Figure 2.
Figure 2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

- If $S_T < X + C_0$, then: call buyer profit $< 0 <$ call seller profit
- If $S_T = X + C_0$, then: call buyer profit $= 0 =$ call seller profit
- If $S_T > X + C_0$, then: call buyer profit $> 0 >$ call seller profit

The breakeven price is a very descriptive term that we use for $X + C_0$, or $X +$ premium.

**Figure 2: Profit Diagram for a Call at Expiration**

Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price $X$:

- $P_T = X - S_T$ if $S_T < X$
- $P_T = 0$ if $X < S_T$

or

- $P_T = \max(0, X - S_T)$

For example, an investor writes a put option on a stock with a strike price of $X = 50$. If the stock stays at $50$ or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below $50$, say to $40$, the put option may be exercised with the option holder buying the stock from the market at $40$ and selling it to the put writer at $50$, for a $10$ gain. The writer of the put option
must pay the put price of $50, when it can be sold in the market at only $40, resulting in a $10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 3 illustrates this example, excluding the initial cost of the put and transaction costs. Figure 4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

Figure 3: Put Payoff to Buyer and Seller

Given the “mirror-image quality” that results from the “zero-sum game” nature of options, we often just draw the profit to the buyer as shown in Figure 4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

Figure 4: Put Profit to Buyer

The breakeven price for a put position upon expiration is the exercise price minus the premium paid, $X - P_0$. 

©2016 Kaplan, Inc.
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

Example: Call option

An investor purchases a call option on a stock with an exercise price of $35. The premium is $3.20. Calculate the payoffs and profits for the option owner at expiration for each of the following prices of the underlying stock $S_T$: $25, $30, $35, $40, $45, and $50. Calculate the breakeven price (assuming no transaction costs).

Professor’s Note: All examples ignore transactions costs. If by chance you see them on the exam, you can easily include them by just adding any costs onto the option premium in calculating breakeven or profits.

Answer:

The figure below contains the payoffs and profits from a long call with an exercise price of $35.

Payoff and Profit on a Long Call Option

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>payoff = (\max(0, S_T - X))</th>
<th>profit = payoff – (C_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25</td>
<td>(\max(25 - 35, 0) = 0)</td>
<td>$0 – $3.20 = –$3.20</td>
</tr>
<tr>
<td>$30</td>
<td>(\max(30 - 35, 0) = 0)</td>
<td>$0 – $3.20 = –$3.20</td>
</tr>
<tr>
<td>$35</td>
<td>(\max(35 - 35, 0) = 0)</td>
<td>$0 – $3.20 = –$3.20</td>
</tr>
<tr>
<td>$40</td>
<td>(\max(40 - 35, 0) = 5)</td>
<td>$5 – $3.20 = $1.80</td>
</tr>
<tr>
<td>$45</td>
<td>(\max(45 - 35, 0) = 10)</td>
<td>$10 – $3.20 = $6.80</td>
</tr>
<tr>
<td>$50</td>
<td>(\max(50 - 35, 0) = 15)</td>
<td>$15 – $3.20 = $11.80</td>
</tr>
</tbody>
</table>

As for the breakeven price, we clearly see that it is between $35 and $40 because the profit turns positive between these two strike prices. The calculation is simple:

breakeven price = $35.00 + $3.20 = $38.20

Example: Put option

An investor purchases a put option on a stock with an exercise price of $15. The premium is $1.60. Calculate the payoffs and profits for the option owner at expiration for each of the following prices of the underlying stock $S_T$: $0, $5, $10, $15, $20, and $25. What is the breakeven price?

Answer:

The following table contains the payoffs and profits from a long put with an exercise price of $15.
Payoff and Profit on a Long Put Option

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>payoff = max(0, X − ST)</th>
<th>profit = payoff − P0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td>max(0, $15 − $0) = $15</td>
<td>$15.00 − $1.60 = $13.40</td>
</tr>
<tr>
<td>$5</td>
<td>max(0, $15 − $5) = $10</td>
<td>$10.00 − $1.60 = $8.40</td>
</tr>
<tr>
<td>$10</td>
<td>max(0, $15 − $10) = $5</td>
<td>$5.00 − $1.60 = $3.40</td>
</tr>
<tr>
<td>$15</td>
<td>max(0, $15 − $15) = $0</td>
<td>$0 − $1.60 = −$1.60</td>
</tr>
<tr>
<td>$20</td>
<td>max(0, $15 − $20) = $0</td>
<td>$0 − $1.60 = −$1.60</td>
</tr>
<tr>
<td>$25</td>
<td>max(0, $15 − $25) = $0</td>
<td>$0 − $1.60 = −$1.60</td>
</tr>
</tbody>
</table>

We see that the breakeven price is between $15 and $10 because the profit turns positive between these two strike prices. The formula is simple:

breakeven price = $15.00 − $1.60 = $13.40

These examples illustrate the properties that we have mentioned so far.

• In both cases, the payoffs and profits are linear functions of ST for the regions above and below X.
• The call option has the potential for an infinite payoff and profit because there is no upper limit to ST − X, nor to ST − X − C0.
• The put has an upper payoff, which is X, and the upper limit to the profit is X − P0.

Obviously, an investor who thinks the stock will go up would have the propensity to either buy a call or sell a put. An investor that thinks a stock will go down would be motivated to either sell a call or buy a put. Investors can take more elaborate positions, as we will discuss.

Covered Calls and Protective Puts

LOS 27.a: Compare the use of covered calls and protective puts to manage risk exposure to individual securities.

Professor’s Note: Covered calls and protective puts are the first of the combined positions. They were taught at Level I and Level II and have been tested at Level III. Know them well.

Covered Call

An investor creates a covered call position by buying the underlying security and selling a call option. Covered call writing strategies are used to generate additional portfolio income when the investor believes that the underlying stock price will remain unchanged over the short term. The profit profile for a covered call is given in Figure 5.
At expiration, the following relationships hold for the investor that both buys the stock and sells the call:

\[
\text{profit} = \max(0, S_T - X) + S_T - S_0 + C_0 \\
\text{maximum profit} = X + C_0 - S_0 \\
\text{maximum loss} = S_0 - C_0 \\
\text{breakeven price} = S_0 - C_0 \\
S_0 = \text{initial stock price paid}
\]

Professor’s Note: The CFA text has more than 65 formulas relating to value at expiration, profit, max profit, max loss, and breakeven(s) in this and the subsequent option positions material. Candidates who try to memorize these equations report being very frustrated and find the formulas are not sufficient to solve all of the end-of-chapter questions. The CFA material does not number the equations or in any way denote that memorization is expected.

We use a different approach in our videos and classes. It is mathematically identical to the formulas and consists of a few steps that work for all combined positions:

Calculate the initial investment in the strategy. Sales are a receipt of funds and purchases an expenditure. Therefore, the initial investment can be a net receipt or expenditure. If the underlying is part of the combination, its initial value at the start of the combination must also be included.

Max profit and loss are all found by examining the payoff pattern to determine the underlying price where max gain or loss occurs. At that underlying price, compute the intrinsic value of all positions in the combination, compare this to the initial investment, and the difference is the max profit or loss.

To compute profit or loss for any stated price of the underlying, the same process applies, just use the specified price of the underlying.

For breakeven start from the max gain or loss and from the payoff pattern, determine if the underlying must increase or decrease and by how much.
Example: Covered call

An investor purchases a stock for $S_0 = 43$ and sells a call for $C_0 = 2.10$ with a strike price, $X = 45$.

1. Show the expression for profit and compute the maximum profit and loss and the breakeven price.

2. Compute the profits when the stock price is $0, 35, 40, 45, 50, \text{ and } 55$.

Answer (1):

\[
\text{profit} = -\max(0, S_T - X) + S_T - S_0 + C_0 \\
\text{maximum profit} = X + C_0 - S_0 \\
\text{maximum loss} = S_0 - C_0 \\
\text{breakeven price} = S_0 - C_0
\]

\[
\begin{align*}
\text{profit} &= -\max(0, S_T - X) + S_T - S_0 + C_0 \\
\text{maximum profit} &= X + C_0 - S_0 \\
\text{maximum loss} &= S_0 - C_0 \\
\text{breakeven price} &= S_0 - C_0
\end{align*}
\]

Answer (2):

The figure below shows profit calculations at the various stock prices.

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Covered Call Profits</th>
<th>profit = $-\max(0, S_T - X) + S_T - S_0 + C_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$-\max(0, 0 - 45) + 0 - 43.00 + 2.10$</td>
<td>$=-40.90$</td>
</tr>
<tr>
<td>$35$</td>
<td>$-\max(0, 35 - 45) + 35.00 - 43.00 + 2.10$</td>
<td>$=-5.90$</td>
</tr>
<tr>
<td>$40$</td>
<td>$-\max(0, 40 - 45) + 40.00 - 43.00 + 2.10$</td>
<td>$=-0.90$</td>
</tr>
<tr>
<td>$45$</td>
<td>$-\max(0, 45 - 45) + 45.00 - 43.00 + 2.10$</td>
<td>$=4.10$</td>
</tr>
<tr>
<td>$50$</td>
<td>$-\max(0, 50 - 45) + 50.00 - 43.00 + 2.10$</td>
<td>$=4.10$</td>
</tr>
<tr>
<td>$55$</td>
<td>$-\max(0, 55 - 45) + 55.00 - 43.00 + 2.10$</td>
<td>$=4.10$</td>
</tr>
</tbody>
</table>

The characteristics of a covered call are that the sale of the call adds income to the position at a cost of limiting the upside gain. It is an ideal strategy for an investor who thinks the stock will neither go up nor down in the near future. As long as the $S_T > S_0 - C_0$ ($40.90$ in the preceding example), the investor benefits from the position.
Study Session 15  
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

**Protective Put**

A protective put (also called portfolio insurance or a hedged portfolio) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, $P_0$. You will see by the diagram in Figure 6 that the investor will still be able to benefit from increases in the stock’s price, but it will be lower by the amount paid for the put, $P_0$. The profit profile for a protective put is shown in Figure 6.

**Figure 6: Protective Put**

Professor’s Note: Economists love these types of graphs because they were forced to spend long hours learning to understand them. Now they like to share that pain. If you like the graphs, use them. *The bottom line is you need to know the net payoff graph shape, the blue line for each combination. All the other lines are interim steps used for constructing the graph. You can skip the other lines since they are not covered in the CFA text. Some candidates like seeing the extra lines.*

At expiration, the following relationships hold:

- **profit** = \( \max(0, X - S_T) + S_T - S_0 - P_0 \)
- maximum profit = \( S_T - S_0 - P_0 \) (no upside limit)
- maximum loss = \( S_0 - X + P_0 \)
- breakeven price = \( S_0 + P_0 \)
Example: Protective put

An investor purchases a stock for $S_0 = 37.50 and buys a put for $P_0 = 1.40 with a strike price, $X = 35$.

(1) **Demonstrate** the expressions for the profit and the maximum profit and **compute** the maximum loss and the breakeven price.

(2) **Compute** the profits for when the price is $0, $30, $35, $40, $45, and $50.

**Answer (1):**

\[
\text{profit} = \max(0, X - S_T) + S_T - S_0 - P_0 \\
\text{maximum profit} = S_T - S_0 - P_0 \\
\text{maximum loss} = S_0 - X + P_0 \\
\text{breakeven price} = S_0 + P_0 = 37.50 + 1.40 = 38.90
\]

**Answer (2):**

The figure below shows profit calculations for the protective put.

### Protective Put Profits

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Protective Put Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$\max(0, 35 - 0) + 0 - 37.5 - 1.40 = -3.90$</td>
</tr>
<tr>
<td>$30$</td>
<td>$\max(0, 35 - 30) + 30.00 - 37.5 - 1.40 = -3.90$</td>
</tr>
<tr>
<td>$35$</td>
<td>$\max(0, 35 - 35) + 35.00 - 37.5 - 1.40 = -3.90$</td>
</tr>
<tr>
<td>$40$</td>
<td>$\max(0, 35 - 40) + 40.00 - 37.5 - 1.40 = 1.10$</td>
</tr>
<tr>
<td>$45$</td>
<td>$\max(0, 35 - 45) + 45.00 - 37.5 - 1.40 = 6.10$</td>
</tr>
<tr>
<td>$50$</td>
<td>$\max(0, 35 - 50) + 50.00 - 37.5 - 1.40 = 11.10$</td>
</tr>
</tbody>
</table>

The characteristics of a protective put are that the purchase of the put provides a lower limit to the position at a cost of lowering the possible profit (i.e., the gain is reduced by the cost of the insurance). It is an ideal strategy for an investor who thinks the stock may go down in the near future, yet the investor wants to preserve upside potential.

**Discussion**

The answers here are per one unit of each asset (e.g., one share of stock and one option). The final results can just be multiplied by the number of units involved. For example, in the preceding protective put example, if an investor had 200 shares of the stock and 200 puts, a value of $S_T = 50$ would give a total profit of $200 \times 11.10$ or $2,220$. 

©2016 Kaplan, Inc.
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

OPTION SPREAD STRATEGIES

LOS 27.b: Calculate and interpret the value at expiration, profit, maximum profit, maximum loss, breakeven underlying price at expiration, and general shape of the graph for the following option strategies: bull spread, bear spread, butterfly spread, collar, straddle, box spread.

Bull Spread

A bull spread provides limited upside if the underlying rises (hence the name bull) with limited downside. It can be constructed when the buyer of the spread purchases a call option with a low exercise price, $X_L$, and subsidizes the purchase price of that call by selling a call with a higher exercise price, $X_H$. The prices are $C_L$ and $C_H$, respectively. At inception, the following relationships hold:

- $X_L < X_H$
- $C_L > C_H$

It is usually the case that $S_0 < X_L$ and almost always that $S_0 < X_H$. The investor who buys a bull spread expects the stock price to rise and the purchased call to finish in-the-money such that $X_L < S_T$. However, the investor does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call. The profit/loss diagram of a bull spread is shown in Figure 7.

Figure 7: Bull Spread Using Calls

$\text{Profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_L + C_H$

maximum profit = $X_H - X_L - C_L + C_H$

maximum loss = $C_L - C_H$

breakeven price = $X_L + C_L - C_H$

breakeven price = $X_L + C_L - C_H$
Example: Bull spread

An investor purchases a call for $C_{L0} = 2.10$ with a strike price of $X = 45$ and sells a call for $C_{H0} = 0.50$ with a strike price of $X = 50$.

(1) Demonstrate the expression for the profit and compute the maximum profit and loss and the breakeven price.

(2) Compute the profits for when the price is $0, 35, 45, 48, 50, \text{ and } 55$.

Answer (1):

\[
\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} \\
= \max(0, S_T - 45) - \max(0, S_T - 50) - 2.10 + 0.50 \\
= \max(0, S_T - 45) - \max(0, S_T - 50) - 1.60 \\
\]

maximum profit \(= X_H - X_L - C_{L0} + C_{H0} \)
\[= 50.00 - 45.00 - 2.10 + 0.50 \]
\[= 3.40 \]

maximum loss \(= C_{L0} - C_{H0} \)
\[= 2.10 - 0.50 \]
\[= 1.60 \]

breakeven price \(= X_L + C_{L0} - C_{H0} \)
\[= 45.00 + 2.10 - 0.50 \]
\[= 46.60 \]

Answer (2):

The following figure shows the calculations of the profit on the bull spread.

### Bull Spread Profits

<table>
<thead>
<tr>
<th>(S_T)</th>
<th>(\text{Bull Spread Strategy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td>(\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} )</td>
</tr>
<tr>
<td>$35</td>
<td>(\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} )</td>
</tr>
<tr>
<td>$45</td>
<td>(\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} )</td>
</tr>
<tr>
<td>$48</td>
<td>(\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} )</td>
</tr>
<tr>
<td>$50</td>
<td>(\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} )</td>
</tr>
<tr>
<td>$55</td>
<td>(\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0} )</td>
</tr>
</tbody>
</table>
The characteristics of a long bull spread (long low exercise call and short high exercise call) are that it provides a potential gain if the stock increases in price but at a lower cost than the single lower exercise-price call alone. The upper limit is capped, however, which is the price of lowering the cost. It is obviously a strategy for an investor with an expectation of the stock’s price increasing in the near term.

**Bear Spread**

A bear spread provides limited upside if the underlying declines (hence the name bear) with limited downside. It is most commonly constructed by selling a call with a low strike price and purchasing a call with a high strike price. As stock prices fall, you keep the premium from the written call, net of the long call premium. The purpose of the long call is to protect you from sharp increases in stock prices. The payoff/profits, shown in Figure 8, are the opposite (inverted image) of the bull spread.

**Figure 8: Bear Spread Using Calls**

![Figure 8: Bear Spread Using Calls](image)

**Bear Spread Using Puts**

Virtually any payoff pattern can be constructed in more than one way. For example, the bear spread can also be constructed using puts. The investor buys a put with the higher exercise price and sells a put with a lower exercise price. The important relationships are:

\[
\begin{align*}
\text{profit} &= \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0} \\
\text{maximum profit} &= X_H - X_L - P_{H0} + P_{L0} \\
\text{maximum loss} &= P_{H0} - P_{L0} \\
\text{breakeven price} &= X_H + P_{L0} - P_{H0}
\end{align*}
\]

**Example: Bear spread using puts**

An investor purchases a put for \( P_{H0} = $4.00 \) with a strike price of \( X_H = $25.00 \) and sells a put for \( P_{L0} = $1.80 \) with a strike price of \( X_L = $20.00 \).

1. Demonstrate the expression for profit and compute the maximum profit and loss and the breakeven price.

2. Calculate the profits when the price is $0, $15, $20, $22, $25, and $30.
Answer (1):

\[
\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}
\]

\[
= \max(0, 25 - S_T) - \max(0, 20 - S_T) - 4.00 + 1.80
\]

\[
= \max(0, 25 - S_T) - \max(0, 20 - S_T) - 2.20
\]

\[
\text{maximum profit} = X_H - X_L - P_{H0} + P_{L0}
\]

\[
= 25.00 - 20.00 - 4.00 + 1.80
\]

\[
= 2.80
\]

\[
\text{maximum loss} = P_{H0} - P_{L0}
\]

\[
= 4.00 - 1.80 = 2.20
\]

\[
\text{breakeven price} = X_H + P_{L0} - P_{H0}
\]

\[
= 25.00 + 1.80 - 4.00
\]

\[
= 22.80
\]

Answer (2):

The following figure shows the calculations of the profits on the bear spread.

<table>
<thead>
<tr>
<th>( S_T )</th>
<th>Bear Spread Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \max(0, 25 - 0) - \max(0, 20 - 0) - 4.00 + 1.80 = 2.80 )</td>
</tr>
<tr>
<td>15</td>
<td>( \max(0, 25 - 15) - \max(0, 20 - 15) - 4.00 + 1.80 = 2.80 )</td>
</tr>
<tr>
<td>20</td>
<td>( \max(0, 25 - 20) - \max(0, 20 - 20) - 4.00 + 1.80 = 2.80 )</td>
</tr>
<tr>
<td>22</td>
<td>( \max(0, 25 - 22) - \max(0, 20 - 22) - 4.00 + 1.80 = 0.80 )</td>
</tr>
<tr>
<td>25</td>
<td>( \max(0, 25 - 25) - \max(0, 20 - 25) - 4.00 + 1.80 = -2.20 )</td>
</tr>
<tr>
<td>30</td>
<td>( \max(0, 25 - 30) - \max(0, 20 - 30) - 4.00 + 1.80 = -2.20 )</td>
</tr>
</tbody>
</table>

Just like the bull spread, this strategy will benefit for the predicted market move (in this case, a prediction of a down movement), but the gain is limited. The cost is lower, however, because of the sold put with the lower exercise price.

Butterfly Spread With Calls

A butterfly spread with calls involves the purchase or sale of four call options of three different types:

- Buy one call with a low exercise price (\( X_L \)).
- Buy another call with a high exercise price (\( X_H \)).
- Write two calls with an exercise price in between (\( X_M \)).
Typically the strike prices are equidistant apart, for example 10, 15, and 20.

The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large. The two graphs in Figure 9 illustrate the construction and behavior of a butterfly spread. The top graph shows the profits of the components, and the bottom graph illustrates the spread itself.

**Figure 9: Butterfly Spread Construction and Behavior**

\[
\begin{align*}
\text{profit} &= \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0} \\
\text{maximum profit} &= X_M - X_L - C_{L0} + 2C_{M0} - C_{H0} \\
\text{maximum loss} &= C_{L0} - 2C_{M0} + C_{H0} \\
\text{breakeven prices} &= X_L + C_{L0} - 2C_{M0} + C_{H0} \text{ and } 2X_M - X_L - C_{L0} + 2C_{M0} - C_{H0}
\end{align*}
\]
Example: Butterfly spread with calls

An investor makes the following transactions in calls on a stock:

- Buys one call defined by \( C_{L0} = $7 \) and \( X_L = $55 \).
- Buys one call defined by \( C_{H0} = $2 \) and \( X_H = $65 \).
- Sells two calls defined by \( C_{M0} = $4 \) and \( X_M = $60 \).

1. Demonstrate the expressions for the profit and the maximum profit and compute the maximum loss and the breakeven price.

2. Calculate the profits for when the price is $50, $55, $58, $60, $62, and $65.

**Answer (1):**

\[
\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0} \\
\]

\[
= \max(0, S_T - 55) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 7 + 2(4) - 2 \\
= \max(0, S_T - 55) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 \\
\]

\[
\text{maximum profit} = X_M - X_L - C_{L0} + 2C_{M0} - C_{H0} \\
= 60 - 55 - 7 + 2(4) - 2 \\
= 4 \\
\]

\[
\text{maximum loss} = C_{L0} - 2C_{M0} + C_{H0} \\
= 7 - 2(4) + 2 = 1 \\
\]

\[
\text{breakeven prices} = 55 + 7 - 2(4) + 2 \text{ and } 2(60) - 55 - 7 + 2(4) - 2 = \text{ $56 \ and \ $64} \\
\]

**Answer (2):**

The figure shows the calculations of the profits on the butterfly spread.

**Butterfly Spread Profits**

<table>
<thead>
<tr>
<th>( S_T )</th>
<th>Butterfly Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>( \max(0, S_T - 55) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 = -1 )</td>
</tr>
<tr>
<td>$55</td>
<td>( \max(0, S_T - 55) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 = -1 )</td>
</tr>
<tr>
<td>$58</td>
<td>( \max(0, S_T - 58) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 = 2 )</td>
</tr>
<tr>
<td>$60</td>
<td>( \max(0, S_T - 60) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 = 4 )</td>
</tr>
<tr>
<td>$62</td>
<td>( \max(0, S_T - 62) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 = 2 )</td>
</tr>
<tr>
<td>$65</td>
<td>( \max(0, S_T - 65) - 2\max(0, S_T - 60) + \max(0, S_T - 65) - 1 = -1 )</td>
</tr>
</tbody>
</table>
Butterfly Spread With Puts

A butterfly spread with puts is constructed by buying one put with a low exercise price, buying a second put with a higher exercise price, and selling two puts with an intermediate exercise price. The profit function is very similar to that of the butterfly spread with calls. You will notice that in each of the max( ) functions, the $S_T$ and $X_i$ have switched, but otherwise it is basically the same format:

$$
\text{profit} = \max(0, X_L - S_T) - 2\max(0, X_M - S_T) + \max(0, X_H - S_T) - P_L + 2P_M - P_H
$$

As with the butterfly spread with calls, the long butterfly spread with puts will have its highest terminal value if the stock finishes at the exercise price for the written puts.

**Example: Butterfly spread with puts**

An investor composes a butterfly spread by buying puts with premiums of $0.80 and $5.50 and exercise prices of $40 and $50, respectively. The investor sells two puts with a premium of $3 and an exercise price of $45. Calculate the profit if the value of the underlying stock at expiration is $46.30.

**Answer:**

$$
\text{profit} = \max(0, $40.00 - $46.30) - 2\max(0, $45.00 - $46.30) + \max(0, $50.00 - $46.30) - ($0.80 + 2($3.00) - $5.50)
$$

$\text{profit} = 0 - 0 + $1.70 - $0.30 = $1.40$

The obvious motivation for the butterfly spread is to earn a profit if the underlying asset does not move very much over the lives of the options used to create the spread. If there is a big movement, then the loss is limited to a lower bound (e.g., $-1$ in the first example and $-0.30$ in the butterfly put example—try it and see). Of course, an investor who thinks there will be a big move will take the other side or short the butterfly spread. The butterfly spread’s appeal is that it limits the loss to the long side of the strategy.

Straddle

A *straddle* consists of the purchase of both a put option and a call option on the same asset. The put and call are purchased with the same exercise price and expiration. In a straddle, you expect a large stock price move, but you are unsure of the direction. You lose if the stock price remains unchanged. The profit/loss diagram for a straddle is shown in Figure 10.
Note that to break even on a straddle, the stock price must move enough to recoup the premiums paid for the options. The breakeven price is equal to the exercise price ± (put + call premium), denoted by points A and B in Figure 10.

For the straddle, the important relationships are:

\[
\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0
\]

maximum profit = \(S_T - X - C_0 - P_0\) (unlimited upside as \(S_T\) increases)

maximum loss = \(C_0 + P_0\)

breakeven price = \(X - C_0 - P_0\) and \(X + C_0 + P_0\)

Example: Straddle

An investor purchases a call on a stock, with an exercise price of $45 and premium of $3, and a put option with the same maturity that has an exercise price of $45 and premium of $2.

(1) Demonstrate the expressions for the profit and the maximum profit and compute the maximum loss and the breakeven price.

(2) Compute the profits when the price is $0, $35, $40, $45, $50, $55, and $100.

Answer (1):

\[
\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0
\]

\[
= \max(0, S_T - $45) + \max(0, $45 - S_T) - $3 - $2
\]

\[
= \max(0, S_T - $45) + \max(0, $45 - S_T) - $5
\]

maximum profit = \(S_T - X - C_0 - P_0\)

\[
= S_T - $45 - $5
\]

maximum loss = \(C_0 + P_0 = $5\)

breakeven price = \(X - C_0 - P_0\) and \(X + C_0 + P_0\)

\[
= $45 - $5 and $45 + $5
\]

\[
= $40 and $50
\]
Answer (2):

The figure below shows the calculation for the profit on a straddle.

**Profits on a Long Straddle**

<table>
<thead>
<tr>
<th>( S_T )</th>
<th>Straddle profit = ( \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td>( \max(0, 0 - 45) + \max(0, 45 - 0) - 3 - 2 = 40 )</td>
</tr>
<tr>
<td>$35</td>
<td>( \max(0, 35 - 45) + \max(0, 45 - 35) - 5 = 5 )</td>
</tr>
<tr>
<td>$40</td>
<td>( \max(0, 40 - 45) + \max(0, 45 - 40) - 5 = 0 )</td>
</tr>
<tr>
<td>$45</td>
<td>( \max(0, 45 - 45) + \max(0, 45 - 45) - 5 = 0 )</td>
</tr>
<tr>
<td>$50</td>
<td>( \max(0, 50 - 45) + \max(0, 45 - 50) - 5 = 0 )</td>
</tr>
<tr>
<td>$55</td>
<td>( \max(0, 55 - 45) + \max(0, 45 - 55) - 5 = 5 )</td>
</tr>
<tr>
<td>$100</td>
<td>( \max(0, 100 - 45) + \max(0, 45 - 100) - 5 = 50 )</td>
</tr>
</tbody>
</table>

The values \( S_T = 0 \) and \( S_T = 100 \) were included in the previous example to illustrate the fact that the upside potential for a long straddle is unlimited, and the downside risk is only the sum of the premiums of the call and put.

---

**Professor’s Note:** It is entirely possible you will get an exam question that “extends the basics.” For example, if a straddle is long the call and put (with the same strike price), a reverse straddle is short the call and put. You initially receive the premiums and profit if the underlying does not move from the strike price. The graph flips on the horizontal axis and looks like a broad, flattened A, instead of a V.

### Collar

A collar is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a zero-cost collar. The usual practice is to select strike prices such that put strike < call strike. Because this is the case, we can continue to use our \( X_L \) and \( X_H \) notation where \( X_L \) is the put strike price and \( X_H \) is the call strike price.

As Figure 11 illustrates, this effectively puts a band or collar around the possible returns. Both the upside and downside are limited, the downside by the long put and the upside by the short call. Many possibilities exist. By lowering \( X_L \), for example, the put premium will fall, so the investor could sell a call with a higher \( X_H \) to offset the lower put premium. With a lower \( X_L \) and higher \( X_H \), the upside and downside potential both increase.
Figure 11: Collar

**Professor’s Note:** The CFA text only covers zero cost collars where the initial net option premium is zero.

---

**Example: Zero-cost collar**

An investor purchases a stock for $29 and a put for $0.20 with a strike price of $27.50. The investor sells a call for $0.20 with a strike price of $30.

1. **Demonstrate** the expression for the profit and **calculate** the maximum profit and loss and the breakeven price.

2. **Calculate** the profits when the price is $0, $20.00, $25.00, $28.50, $30.00, and $100.00.

**Answer (1):** This is a zero-cost collar because the premiums on the call and put are equal.

\[
\text{profit} = \max(0, X_L - S_T) - \max(0, S_T - X_H) + S_T - S_0
\]

maximum profit \(= X_H - S_0\)
maximum loss \(= S_0 - X_L\)
breakeven price \(= S_0\)

\[
\text{profit} = \max(0, 27.50 - S_T) - \max(0, S_T - 30.00) + S_T - 29.00
\]

maximum profit \(= 30.00 - 29.00 = 1\)
maximum loss \(= 29.00 - 27.50 = 1.50\)
breakeven price \(= 29.00\)
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

Answer (2):

The table on the following page shows the calculations for profits on this zero-cost collar.

Profits on a Zero-Cost Collar

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Zero-Cost Collar profit = [\max(0, X_L - S_T) - \max(0, S_T - X_H) + S_T - S_0]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.00$</td>
<td>[\max(0, 27.50 - 0) - \max(0, 0 - 30.00) + 0 - 29.00] = $-1.50</td>
</tr>
<tr>
<td>$20.00$</td>
<td>[\max(0, 27.50 - 20.00) - \max(0, 20.00 - 30.00) + 20.00 - 29.00] = $-1.50</td>
</tr>
<tr>
<td>$25.00$</td>
<td>[\max(0, 27.50 - 25.00) - \max(0, 25.00 - 30.00) + 25.00 - 29.00] = $-1.50</td>
</tr>
<tr>
<td>$28.50$</td>
<td>[\max(0, 27.50 - 28.50) - \max(0, 28.50 - 30.00) + 28.50 - 29.00] = $0.50</td>
</tr>
<tr>
<td>$30.00$</td>
<td>[\max(0, 27.50 - 30.00) - \max(0, 30.00 - 30.00) + 30.00 - 29.00] = $1.00</td>
</tr>
<tr>
<td>$100.00$</td>
<td>[\max(0, 27.50 - 100.00) - \max(0, 100.00 - 30.00) + 100.00 - 29.00] = $1.00</td>
</tr>
</tbody>
</table>

We see how the lower limit of dollar return is $-1.50$, even when the underlying asset’s price is zero. The upper limit on return or profit is $1$, even when the underlying asset’s price is $100$. For a price of the underlying asset between the strike prices, such as $S_T = 28.50$ in this example, the profit is between $-1.50$ and $1.00$. The collar is a good strategy for locking in the value of a portfolio at a minimal cost. The cost is zero if the appropriate put and call have the same premium.

Box Spread Strategy

The box spread is a combination of a bull spread and a bear spread on the same asset, using only two strike prices. For example, a bull spread is the combination of two calls: a short call with a higher strike price ($X_H$) and a long call with a lower strike price ($X_L$).

Figure 12: Bull Spread

The bear spread is a short put with a lower strike price ($X_L$) and long put with a higher strike price ($X_H$).
Combining the two produces the box spread with an interesting result. The ending value of the box spread is the same no matter what the ending value of the underlying. The initial investment (net option premium) is the same, so if the options are priced correctly, the difference in ending and beginning value of the box spread must reflect the risk-free rate. If the options are not priced correctly and the box spread return is not the risk-free rate, the box spread has identified an arbitrage opportunity.

**Example: Box spread**

An investor buys a call and sells a put with a strike price of $X_L = $25. The call and put premiums are $C_{L0} = $1.75 and $P_{L0} = $0.50. The investor then sells a call and buys a put with a strike price of $X_H = $30. For the second pair of options, the call and put premiums are $C_{H0} = $0.20 and $P_{H0} = $3.90. The options all expire in two months.

Compute the profit and the annualized return on the investment and determine whether this a worthwhile investment, if the risk-free rate is 5%.
Answer:

\[
\text{profit} = X_H - X_L + P_{L0} - C_{L0} + C_{H0} - P_{H0}
\]

\[
\text{profit} = $30.00 - $25.00 + $0.50 - $1.75 + $0.20 - $3.90
\]

\[
\text{profit} = $0.05
\]

The initial cost was $4.95 = + $0.50 – $1.75 + $0.20 – $3.90. This means the holding period return is 0.05 / 4.95 = 0.0101. This is a 2-month return, so the annualized return is 0.06216 (≈ 1.0101^{12/2} – 1). Because this return is greater than the risk-free rate of 5%, this would be a worthwhile strategy. If the investor can borrow for less than 0.06216 (6.2%), an arbitrage profit is possible.

**Interest Rate Options**

**LOS 27.c:** Calculate the effective annual rate for a given interest rate outcome when a borrower (lender) manages the risk of an anticipated loan using an interest rate call (put) option.

Professor’s Note: The general rule for interest rate options (such as caps and floors) is the interest rate for the payout is set at the expiration of the option but paid at the end of the interest rate period, not when the option expires.

Hopefully, you are already familiar with the basic mechanics of an interest rate call option that makes a payment to the owner when the reference rate (a.k.a. the underlying) exceeds the strike rate (i.e., the exercise rate). Because LIBOR is the usual reference rate, we will put that in the formula. The formula for the payment is:

\[
\text{payoff} = (NP)\left[\max(0, \text{LIBOR} - \text{strike rate})\right]\left(\frac{D}{360}\right)
\]

where \(NP\) stands for notional principal and \(D\) stands for days in underlying rate (i.e., the number of days the notional principal would be theoretically borrowed).

**NOTE:** Do not confuse this with the maturity of the call! **Maturity** is the time between today and when the payoff is determined.

As an example, we will say that the notional principal of the contract is $20 million, the option expires in 49 days, the strike rate is 6%, and \(D = 90\) days. If at option maturity in 49 days LIBOR is 6.2%, the payoff would be:

\[
\text{payoff} = ($20,000,000)(0.002)(90 / 360) = $10,000
\]

The payoff will occur 139 days after the purchase of the option. If the underlying rate (LIBOR) had been less than 6%, then the payoff would have been zero. Because the call has a positive payoff when interest rates rise above a certain level, they can hedge a floating-rate loan.
Example: Interest rate call option

On March 1, a firm plans to borrow $10 million for 90 days beginning on April 1 (31 days in the future, which is the maturity of the call). It can currently borrow at LIBOR plus 200 basis points, and LIBOR is currently 4.5%. The firm buys an interest rate call option where LIBOR is the underlying, and the strike rate is 4%. The notional principal is $10 million, and \( D = 90 \) days, which is also the length of the loan. The premium of the call is $5,000. Calculate the effective borrowing rates of the loan when LIBOR is 2.0%, 3.5%, 4.0%, 4.5%, and 6%.

Answer:

If the manager chooses to purchase the call, that is a cost today. To accurately measure its effect on the borrowing costs, we need to compute its (future) value at the date of the loan, using the firm’s cost of borrowing (LIBOR + 0.02):

\[
FV(\text{premium}) = \text{premium} \left[1 + (\text{current LIBOR} + \text{spread})(\text{maturity} / 360)\right]
\]

\[
FV(\text{premium}) = \$5,000 \left[1 + (0.045 + 0.02)(31 / 360)\right]
\]

\[
FV(\text{premium}) = \$5,028
\]

Hence, when the firm actually borrows on April 1 (31 days in the future), it is effectively receiving:

\[
\text{net amount} = \text{loan} - FV(\text{premium})
\]

\[
\text{net amount} = \$10,000,000 - \$5,028
\]

\[
\text{net amount} = \$9,994,972
\]

With the call premium now reflected in the net proceeds of the loan, the interest cost will be LIBOR plus the 200 basis point spread at that time less any payoff from the call:

\[
\text{effective dollar interest cost} = \$10,000,000(\text{LIBOR}_{\text{April 1}} + 0.02)(90 / 360) - (\text{call payoff})
\]

The call payoff is:

\[
\text{payoff} = (\text{NP}) \left[\max(0, \text{LIBOR} - \text{strike rate})(D / 360)\right]
\]

\[
\text{payoff} = (\$10,000,000) \left[\max(0, \text{LIBOR}_{\text{April 1}} - 0.04)(90 / 360)\right]
\]

The effective annual rate (EAR) of borrowing for the 90 days is:

\[
\text{EAR} = \left(\frac{\$10,000,000 + \text{effective dollar interest cost}}{\$9,994,972}\right)^{365/90} - 1
\]
Let's look at the two extremes first: 2% and 6%. If LIBOR is less than 4%, the call payoff is zero. If LIBOR is 2% on April 1, for example, the effective dollar interest cost is:

\[ \text{Effective dollar interest cost} = 100,000 = 10,000,000(0.02 + 0.02)(90 / 360) \]

If the firm did not hedge, the effective annual rate would be:

\[
\begin{align*}
\text{EAR without hedge} &= \left( \frac{10,100,000}{10,000,000} \right)^{\frac{365}{90}} - 1 \\
\text{EAR without hedge} &= 0.04118
\end{align*}
\]

Including the cost of the call will increase the rate to:

\[
\begin{align*}
\text{EAR with hedge} &= \left( \frac{10,100,000}{9,994,972} \right)^{\frac{365}{90}} - 1 \\
\text{EAR with hedge} &= 0.04331
\end{align*}
\]

Thus, the cost of the call is incorporated into the effective rate of the loan. Just like a purchased call on a stock, if the underlying is below the strike at expiration, the buyer loses (i.e., the option is worthless and the buyer has paid a premium for it).

Where the borrowing firm benefits is when LIBOR is higher than the strike rate. If LIBOR on April 1 = 6%, the option payoff is:

\[
\text{payoff} = 10,000,000(0.06 - 0.04)(90 / 360) = 50,000
\]

\[
\text{effective dollar interest cost} = 10,000,000(0.06 + 0.02)(90 / 360) - 50,000 = 150,000
\]

Comparing the EAR with and without the hedge:

\[
\begin{align*}
\text{EAR without hedge} &= \left( \frac{10,200,000}{10,000,000} \right)^{\frac{365}{90}} - 1 \\
\text{EAR without hedge} &= 0.08362
\end{align*}
\]

Including the cost of the call will decrease the effective rate to:

\[
\begin{align*}
\text{EAR with hedge} &= \left( \frac{10,150,000}{9,994,972} \right)^{\frac{365}{90}} - 1 \\
\text{EAR with hedge} &= 0.06441
\end{align*}
\]

In fact, this effective rate of 0.06441 is the highest rate the firm can expect to pay with the call.

Let’s look at what happens for the cases in between, where 0.02 < LIBOR on April 1 < 0.06. If LIBOR on April 1 = 0.035, the firm will incur dollar interest costs equal to:

\[
\text{effective dollar interest cost} = 10,000,000(0.035)(90 / 360) = 137,500
\]

because the call expires worthless. The effective rate on the net inflow from the borrowing is:

\[
\text{EAR with hedge} = \left( \frac{10,137,500}{9,994,972} \right)^{\frac{365}{90}} - 1 = 0.05910
\]
If LIBOR_{April 1} = 0.04, the firm will incur dollar interest costs equal to:

\[
effective dollar interest cost = $10,000,000 \times 0.06 \times \frac{90}{360} = $150,000
\]

because the call expires worthless. The effective rate on the net inflow from the borrowing is:

\[
EAR \text{ with hedge} = \frac{($10,150,000 \div $9,994,972)^{\frac{365}{90}} - 1}{0.06441}
\]

If LIBOR_{April 1} = 0.045, the firm will earn a payoff on the call:

\[
\text{payoff} = ($10,000,000) \times \max(0, 0.045 - 0.04) \times \frac{90}{360} = $12,500
\]

The effective dollar interest cost will be:

\[
effective dollar interest cost = $10,000,000(0.065)(\frac{90}{360}) - $12,500
\]

\[
effective dollar interest cost = $162,500 - $12,500 = $150,000
\]

The effective rate on the net inflow from the borrowing is:

\[
EAR \text{ with hedge} = \frac{($10,150,000 \div $9,994,972)^{\frac{365}{90}} - 1}{0.06441}
\]

This is the same effective cost for when LIBOR_{April 1} = 0.06.

Let’s try another example with less explanation in the answer.

**Example: Interest rate option**

In 40 days, a firm wishes to borrow $5 million for 180 days. The borrowing rate is LIBOR plus 300 basis points. The current LIBOR is 5%. The firm buys a call that matures in 40 days with a notional principal of $5 million, 180 days in underlying (D = 180), and a strike rate of 4.5%. The call premium is $8,000.

Calculate the effective annual rate of the loan if at expiration LIBOR = 4%, and calculate if LIBOR = 5%.

**Answer:**

First we compute the implied net amount to be borrowed after the cost of the call:

\[
$5,000,000 - $8,000[1 + (0.05+0.03)(40 \div 360)] = $4,991,929
\]

For LIBOR = 0.04 at expiration, the dollar cost is (the option is out-of-the-money):

\[
$5,000,000(0.07)(180 \div 360) = $175,000
\]
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

The effective annual rate is:

\[
\left( \frac{5,175,000}{4,991,929} \right)^{\left(\frac{365}{180}\right)} - 1 = 0.0758
\]

For LIBOR = 0.05, the call option is in-the-money:

\[
\text{payoff} = (5,000,000) \times \max(0, 0.05 - 0.045) \times \left(\frac{180}{360}\right) = 12,500
\]

The dollar interest cost is effectively:

\[
5,000,000(0.08) \times \left(\frac{180}{360}\right) - 12,500 = 187,500
\]

The effective annual rate is:

\[
\left( \frac{5,187,500}{4,991,929} \right)^{\left(\frac{365}{180}\right)} - 1 = 0.0810
\]

You should verify that the rate of 0.0810 is the highest possible rate by trying other values higher than LIBOR = 4.5%.

**Interest Rate Put**

An interest rate put has a payoff to the owner when the reference rate, usually LIBOR, is below a certain strike rate at the maturity of the option:

\[
\text{payoff} = (NP) \times \max(0, \text{strike rate} - \text{LIBOR}) \times \left(\frac{D}{360}\right)
\]

A lender can combine a long position in an interest rate put with a specific floating-rate loan to place a lower limit on the income to be earned on the position. The combination has many of the same basic mechanics as borrowing with an interest rate call. As in the case of the interest rate call, we compute the future value of the put premium, but we add it to the loan made by the lender because that represents the total outflow of cash from the lender at the time of the loan.

As in the case of the interest rate call, the payoff of the put places a limit on the effective dollar interest. In this case, the payoff is added to the interest received to ensure a minimum amount of revenue to the lender. To make our example easier to follow, we will look at the same loan examined in our last example, which was the second example of an interest rate call. Now we will look at it from the lender’s point of view.

**Example: Interest rate put**

In 40 days, a bank plans to lend $5 million for 180 days. The lending rate is LIBOR plus 300 basis points. The current LIBOR is 5%. The bank buys a put that matures in 40 days with a notional principal of $5 million, 180 days in the underlying, and a strike rate of 4.5%. The put premium is $5,000. **Calculate** the effective annual rate of the loan if at expiration LIBOR = 4%, and then **calculate** the rate if LIBOR = 5%.
Answer:
First we compute the total amount the bank pays out (lends) at time of the loan. This means computing the future value of the premium and adding it to the loan amount.

Loan amount plus future value of premium paid:

\[ \$5,000,000 + \$5,000 \times [1 + (0.05 + 0.03)(40 / 360)] = \$5,005,044 \]

This amount is used for computing the effective interest rate earned on the outflow of cash at the beginning of the loan. The dollar interest earned by the bank will be based upon the prevailing rate applied to the loan and the payoff of the put. In this case, the expression is:

\[
\text{effective interest earned} = \$5,000,000 (\text{LIBOR}_{\text{maturity}} + 0.03)(180 / 360) + \text{put payoff}
\]

The effective annualized rate on the loan is:

\[
\text{EAR} = \left( \frac{\$5,000,000 + \text{effective dollar interest earned}}{\$5,005,044} \right)^{\frac{365}{180}} - 1
\]

You can see where the lender gets hurt because both the principal returned and the interest earned are based upon the $5 million, but the effective loan is $5,005,044.

If LIBOR\textsubscript{maturity} equals 4%, the payoff of the put would be:

\[
\text{payoff} = (\$5,000,000)[\max(0, 0.045 - 0.04)(180 / 360)] = \$12,500
\]

The dollar interest earned is:

\[
\$5,000,000(0.04 + 0.03)(180 / 360) = \$175,000
\]

The effective interest rate is:

\[
\text{EAR} = \left( \frac{\$5,187,500}{\$5,005,044} \right)^{\frac{365}{180}} - 1
\]

\[
\text{EAR} = 0.07531 \text{ or } 7.531\%
\]

While not asked, you might notice that in this case the put turned out to be desirable. Without the put the bank would have earned LIBOR + 300 bp or 7%.

If LIBOR\textsubscript{maturity} = 0.05, the dollar interest earned is:

\[
\$5,000,000[0.05 + 0.03](180 / 360) = \$200,000
\]

\[
\text{EAR} = \left( \frac{\$5,200,000}{\$5,005,044} \right)^{\frac{365}{180}} - 1
\]

\[
\text{EAR} = 0.08057 \text{ or } 8.057\%
\]

Without the hedge, and LIBOR = 5% + 300 bp, the lender would have earned $200,000 on only $5 million for an effective rate of 0.08278 = \left( \frac{\$5,200,000}{\$5,000,000} \right)^{\frac{365}{180}} - 1.
Study Session 15  
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

INTEREST RATE CAPS, FLOORS, AND COLLARS

LOS 27.d: Calculate the payoffs for a series of interest rate outcomes when a floating rate loan is combined with 1) an interest rate cap, 2) an interest rate floor, or 3) an interest rate collar.

An interest rate cap is an agreement in which the cap seller agrees to make a payment to the cap buyer when the reference rate exceeds a predetermined level called the cap strike or cap rate. The cap is a series of interest rate call options. Each individual option can be called a caplet. An interest rate floor is an agreement in which the seller agrees to pay the buyer when the reference rate falls below a predetermined interest rate called the floor strike or floor rate. The floor is a series of interest rate put options. Each individual option can be called a floorlet.

Caps and floors are over-the-counter contracts, so the two parties involved can tailor the agreement to suit their specific needs. Generally, the terms of a cap or floor agreement will include the:

• Reference rate (typically LIBOR).
• Cap or floor strike that sets the ceiling or floor.
• Length of the agreement.
• Reset frequency, which determines days in each settlement period, \( D_t \).
• Notional principal (\( NP \)).

For the Exam:

• The CFA text follows the convention that the payoff on the individual caplets and floorlets in interest rate caps and floors based on LIBOR is for the actual number of days in the interest rate period divided by 360. For example, suppose the annual rate is 8% for a quarterly payment and the actual days in the quarter are 92 days; the periodic rate is 8% \( \times (92 / 360) = 2.04444\% \). If the actual day count had not been given, this could be approximated as 8% / 4 = 2.00%.
• Like individual interest rate calls and puts, the payments are in arrears with the rate at the expiration of each caplet or floorlet determining the payoff at the end of the next interest rate period.
• Floating rate loan interest payments are also set in arrears. At the origination of the loan, the first interest payment is known and no caplet covers the first loan period. Instead, the first caplet expires at the end of the first loan interest period to be paid at the end of the second loan interest period. Think of the first “floating rate” as in fact a fixed rate known at initiation of the loan, only the subsequent payments are unknown and floating.

These conventions are illustrated in the following examples.
Interest Rate Caps

An interest rate cap is a series of call options on interest rates. The buyer receives the interest rate difference if rates are above the strike rate. Each potential payoff is called a caplet. The natural user of a cap is the payer on a floating rate loan.

Example: Interest rate cap

On April 15, KS, Inc., takes out a one-year floating rate loan for $10 million. Interest payments are quarterly at LIBOR plus 200 basis points based on actual days in the period over 360. The payments are due July 15, October 15, January 15, and April 15. KS purchases a nine-month, quarterly pay cap for $15,000 with a strike rate of 8.5%. The first caplet expires July 15.

Assuming LIBOR rates on April 15, July 15, October 15, and January 15 are 8.0%, 8.4%, 8.65%, and 8.4% respectively, determine the four payoff dates on the loan, the loan interest paid, any option payment received, and the effective net interest paid.

Day counts:
- April 15 to July 15: 91 days
- July 15 to October 15: 92 days
- October 15 to January 15: 92 days
- January 15 to April 15: 90 days

Answer:

The payment dates on the loan as stated in the question are July 15, October 15, January 15, and April 15.

Loan interest due:
- July 15: $10,000,000 × (0.08 + 0.02) × (91 / 360) = $252,778
- October 15: $10,000,000 × (0.084 + 0.02) × (92 / 360) = $265,778
- January 15: $10,000,000 × (0.0865 + 0.02) × (92 / 360) = $272,167
- April 15: $10,000,000 × (0.084 + 0.02) × (90 / 360) = $260,000

Cap payoffs:
- July 15: N/A. The loan originates April 15 with first loan payment due July 15 based on LIBOR as of April 15. Because the first loan interest payment is known at initiation of the analysis, the first caplet expires July 15 with payoff (if in the money) on October 15.
- October 15: July 15 start of period LIBOR is 8.4%, below the strike rate of 8.5%, caplet is out of the money.
January 15: October 15 start of period LIBOR is 8.65%, above the strike rate of 8.5%, caplet is in the money: $10,000,000 × (0.0865 – 0.085) × (92 / 360) = $3,833.

April 15: January 15 start of period LIBOR is 8.4%, below the strike rate of 8.5%, caplet is out of the money.

Effective net interest due:
- July 15: $252,778
- October 15: $265,778
- January 15: $272,167 – 3,833 = $268,334
- April 15: $260,000 – 0 = $260,000

When a long position in a cap is combined with a floating-rate loan, the payoffs can offset interest costs when the floating rate increases. Because caps trade over the counter, the terms of the cap are very flexible, so the cap buyer/borrower can align the settlements of the cap with the interest rate payments.

### Interest Rate Floors

An interest rate floor is in essence the opposite of a cap. The buyer receives the interest rate difference if rates are below the strike rate. Each potential payoff is called a floorlet. The natural user of a floor is the receiver on a floating rate loan.

#### Example: Interest rate floor

The facts in the question are the same as in the previous example, except the example is a lender who purchases a floor.

On April 15, DHBank makes a one-year floating rate loan for $10 million. Interest payments are quarterly at LIBOR plus 200 basis points based on actual days in the period over 360. The payments are due July 15, October 15, January 15, and April 15. DHBank purchases a nine-month, quarterly pay floor for $85,000 with a strike rate of 8.5%.

Assuming LIBOR rates on April 15, July 15, October 15, and January 15 are 8.0%, 8.4%, 8.65%, and 8.4%, respectively, determine the four payoff dates on the loan, the loan interest received, any option payment received, and the effective net interest earned by DHBank.

Day counts:
- April 15 to July 15: 91 days
- July 15 to October 15: 92 days
- October 15 to January 15: 92 days
- January 15 to April 15: 90 days
**Answer:**

The payment dates on the loan as stated in the question are July 15, October 15, January 15, and April 15.

**Loan interest due:**

- July 15: $10,000,000 \times (0.08 + 0.02) \times (91 / 360) = $252,778
- October 15: $10,000,000 \times (0.084 + 0.02) \times (92 / 360) = $265,778
- January 15: $10,000,000 \times (0.0865 + 0.02) \times (92 / 360) = $272,167
- April 15: $10,000,000 \times (0.084 + 0.02) \times (90 / 360) = $260,000

**Floor payoffs:**

- July 15: N/A. The loan originates April 15 with first loan payment due July 15 based on LIBOR as of April 15. Because the first loan interest payment is known at initiation of the analysis, the first floorlet expires July 15 with payoff (if in the money) on October 15.
- October 15: July 15 start of period LIBOR is 8.4%, below the strike rate of 8.5%, floorlet is in the money: $10,000,000 \times (0.085 – 0.084) \times (92 / 360) = $2,556.
- January 15: October 15 start of period LIBOR is 8.65%, above the strike rate of 8.5%, floorlet is out of the money.
- April 15: January 15 start of period LIBOR is 8.4%, below the strike rate of 8.5%, floorlet is in the money: $10,000,000 \times (0.085 – 0.084) \times (90 / 360) = $2,500

**Effective net interest due:**

- July 15: $252,778
- October 15: $265,778 + 2,556 = $268,334
- January 15: $272,167 + 0 = $272,167
- April 15: $260,000 + 2,500 = $262,500

**Interest Rate Collar**

An interest rate collar is a combination of a cap and a floor where the agent is long in one position and short in the other. If the agent buys a 6% cap on LIBOR and sells a 3% floor on LIBOR, the agent will receive cash payments when LIBOR exceeds 6%, and the agent will make payments when LIBOR is below 3%. If LIBOR is between 3% and 6%, the agent neither receives nor pays.

This would be attractive to a bank that has among its liabilities large deposits with floating interest rates. When the rates start to rise, the bank's increasing costs can be offset by the payments from the collar. By selling the floor, the bank may have to make payments if the interest rates on the deposits fall too much, but the bank earned a premium for exposing itself to this risk. That premium offsets the cost of the cap. The overall position provides some certainty to the bank, because it essentially provides a predetermined range for the cost of funds.
A special interest rate collar occurs when the initial premiums on the cap and the floor are equal and offset each other. Suppose that the premium on a 4-year, 3% floor is equal to the premium on the 6% cap. The combination of the two would be called a zero-cost collar (a.k.a. a zero-premium collar). The motivation for zero-cost collars is that they are a way of providing interest rate protection without the cost of the premiums. Calling the collar zero cost is misleading in some regards. There is no initial cost but there is a back end cost if rates move in such a way that payments must be made.

Example: Interest rate collar

On December 15, the DHBank issues a $50 million “two-year” floating rate liability. The four interest payments are based on 180-day LIBOR plus 150 basis points. The first interest rate is set today with payment 180 days thereafter. Each loan payment is 180 days after the preceding payment. To hedge against rising interest rates, the bank buys an appropriate interest rate cap with a strike rate of 4.75%. To fully offset the initial cost of the cap, the bank sells a floor with a strike rate of 2.25%.

Initial LIBOR is 3.40%. Assuming that in 180, 360, 540, and 720 days LIBOR rates are 4.00%, 5.10%, 2.00%, and 1.75%, respectively, calculate the net interest paid by the bank on each payment date and show all the cash flows leading to that net payment. Explain the cost of the collar.

Answer:

Payment in 180 days:
- Paid on floating rate liability: $50,000,000 × (0.034 + 0.015) × (180/360) = $1,225,000
- No payments on the cap or floor. The first of three caplets and floorlets will expire and possible payments will be determined for the next loan payment date.
- Net paid on loan: $1,225,000

Payment in 360 days:
- Paid on floating rate liability: $50,000,000 × (0.040 + 0.015) × (180/360) = $1,375,000
- The cap is out of the money with beginning of period LIBOR at 4.00% versus a cap strike rate of 4.75%
- The floor is out of the money with beginning of period LIBOR at 4.00% versus a floor strike rate of 2.25%
- Net paid on loan: $1,375,000 – 0 + 0 = $1,375,000
Payment in 540 days:

Paid on floating rate liability: $50,000,000 \times (0.051 + 0.015) \times (180 / 360) = $1,650,000

The cap is in the money with beginning of period LIBOR at 5.10% versus a cap strike rate of 4.75%: Receive $50,000,000 \times (0.051 – 0.0475) \times (180 / 360) = $87,500

The floor is out of the money with beginning of period LIBOR at 5.10% versus a floor strike rate of 2.25%

Net paid on loan: $1,650,000 – 87,500 + 0 = $1,562,500

Payment in 720 days:

Paid on floating rate liability: $50,000,000 \times (0.020 + 0.015) \times (180 / 360) = $875,000

The cap is out of the money with beginning of period LIBOR at 2.00% versus a cap strike rate of 4.75%

The floor is in the money with beginning of period LIBOR at 2.00% versus a floor strike rate of 2.25%: Pay $50,000,000 \times (0.0225 – 0.0200) \times (180 / 360) = $62,500

Net paid on loan: $875,000 – 0 + 62,500 = $937,500

For a properly structured collar, there will be no additional caplet or floorlet expirations on or after the liability due date. The LIBOR rate at day 720 is irrelevant.

The true cost of the collar is not zero even though the net initial premium was zero. Any time the floor is in the money the bank incurs an obligation to pay on the floor that was sold. For example, on day 720, the bank must make a payment on the floor of $62,500. This increases the bank’s cost of funds.

Discussion

Caps and floors are the most common way to modify interest payments or receipts because they can be tailored for exact dates, amounts, and day count conventions. All else the same, the selection of strike rates determines the degree of protection that can be obtained and also affects the premium paid. Like any option, the more in the money the option is at initial purchase, the higher the premium paid and the more likely there will be subsequent payoffs; the more out of the money the option is at initial purchase, the lower the premium paid and the less likely there will be subsequent payoffs. Interest payments can also be modified with FRAs, interest rate swaps, and interest rate futures. These have no initial premium but generally modify upside and downside symmetrically while caps and floors (options) allow for tailored, asymmetric payoff patterns.
Delta Hedging

LOS 27.e: Explain why and how a dealer delta hedges an option position, why delta changes, and how the dealer adjusts to maintain the delta hedge.


delta hedging a derivative position means combining the option position with a position in the underlying asset to form a portfolio, whose value does not change in reaction to changes in the price of the underlying over a short period of time. The value of that portfolio should grow at the risk-free rate over time, as it is dynamically managed.

Dealers in derivatives stand ready to sell an option, such as a call, for which they earn a fee. If they sell the call naked (i.e., without any offsetting position), they would want to form a hedge to limit the risk. The dealer could hedge a short call by buying a call and effectively closing the position. The most popular activity, however, is to delta hedge a naked call by owning the underlying asset in an amount that will make the value of the short-call/long-asset portfolio immune to changes in the price of the underlying.

You may recall the covered-call strategy where an investor sells one call for each share of stock held. At expiration, the profit of a covered-call portfolio had an upside limit. Delta hedging differs because it focuses on the value of the call and how it changes prior to expiration. As it turns out, in delta hedging, the dealer can own fewer shares of stock (or units of the underlying asset) than the number of calls; that proportion is the delta. The symbol for delta is $\Delta$, and it is a commonly used symbol for change.

$\Delta$, for our purposes here, is the change in the price of an option for a 1-unit change in the price of the underlying security. For a call, a simple representation is:

$$\delta_{\text{call}} = \frac{C_1 - C_0}{S_1 - S_0} = \frac{\Delta C}{\Delta S}$$

where:

$\delta_{\text{call}} = \delta$ of the call option

$\Delta C = \text{change in the price of the call over a short time interval}$

$\Delta S = \text{change in the price of the underlying stock over a short time interval}$

As the following examples will show, for a call: $0 \leq \delta \leq 1$. (For long puts and short calls, delta fluctuates between 0 and –1.) We will demonstrate why that property means that it takes fewer shares of stock to hedge a given number of calls. Also, we will see what makes delta increase or decrease in value.
Example: Calculating delta

During the last 20 minutes of trading, shares of a stock have risen from $51.30 to $52.05, and a given call option on the stock has risen from $1.20 to $1.60 in price. Calculate the delta of the call option.

Answer:

The call option delta can be estimated as:

\[
delta_{\text{call}} = \frac{C_1 - C_0}{S_1 - S_0} = \frac{\Delta C}{\Delta S}
\]

\[
delta_{\text{call}} = \frac{$1.60 - $1.20}{$52.05 - $51.30} = \frac{$0.40}{$0.75} = 0.533
\]

We can also calculate the change in option price given the option delta and the change in the security's price by rearranging the formula for the option delta as follows:

\[
\Delta C = \Delta S \times \delta_{\text{call}}
\]

The latter formula is usually of more concern to the dealer. This is because the dealer wants to estimate how the option price will change in response to a change in the price of the underlying. This is an estimation of the amount of risk in the position. The following is a simple demonstration.

Example: Using delta to estimate change in price

Call options on the stock in the previous example have a delta of 0.533. If the stock price rises by $1.50, calculate the approximate change in the price of the option.

Answer:

We can expect the call option to rise by $0.80:

\[
\Delta C = 0.533 \times $1.50 = $0.80
\]

For an investor or dealer who is short the call, this means an increased liability of $0.80 per call.

The question is how a dealer who is short the call can hedge the risk of the position.
Example: Delta hedge

Call options on the stock in the previous example have a delta of 0.533. A dealer is short 100 contracts or 10,000 calls. Calculate the number of shares of the stock to delta hedge this portfolio.

Answer:

If the stock goes up by $1, it means each call goes up by $1(0.533) = $0.533, which is an increased liability to the dealer, but the dealer can hedge each call with 0.533 shares of stock. That is, owning 10,000(0.533) = 5,330 shares of stock will provide a delta-hedged call position. The reason is obvious: if the stock price goes up by $1, then the value of the stock position goes up by $5,330, while the liability of the short call will increase by the same amount and the two will immediately offset each other.

Unfortunately, it is not that easy for three very important reasons:

• Delta is only an approximation of the relative price changes of the stock and call and is less accurate for larger changes in stock price, \( \Delta S \).
• Delta changes as market conditions change, including changes in \( S \).
• Delta changes over time without any other changes.

Professor’s Note: The main challenge of delta hedging is to constantly adjust the hedge to accommodate changes in \( S \) and the passage of time. You must constantly rebalance the hedge for the new delta. On the exam you could be given a table with delta at various stock prices or dates and need to select the delta to use. You might have to calculate delta from option and underlying prices as in the previous example. You should also remember that if you are given Black-Scholes data the \( N(d_1) \) term is the delta of the option. Finally remember that the absolute value of the call and put sum to 1.00. If you were given a call delta of .25 then the put with same expiration and strike delta is -.75.

Because we are hedged, the portfolio should be earning a risk-free rate of return based on a gain in value of the short call position and the fall in value of the stock.

Example: Delta hedge

The initial value of a call is $1.40, which has a delta of 0.5739 as calculated in the Black-Scholes option pricing model and 30 days to maturity. A dealer sells 200 contracts or 20,000 calls. To delta hedge the position, the dealer purchases 20,000(0.5739) = 11,478 shares of the stock at $100/share. A day later, the price of the stock is the same with a new call price of $1.39 and a delta of 0.5727. Calculate the initial value of the position and its value on the next day.
Answer:

Because the call is a liability (the dealer sold the call contracts), we subtract the value of the position from the value of the 11,478 shares:

\[
\text{initial value of portfolio} = \$100(11,478) - \$1.40(20,000) = \$1,119,800
\]

A day later we find:

\[
\text{next-day value of portfolio} = \$100(11,478) - \$1.39(20,000) = \$1,120,000
\]

The value of the portfolio has increased by 0.01785% \((200 / \$1,119,800)\). This shows how the value of the portfolio will increase over time if the other inputs do not change. If precisely measured (i.e., without rounding), the rate of growth per day would be commensurate with the risk-free rate for one day at 6%: \(e^{0.06/365} - 1 = 0.000164\), or 0.0164%.

As a point of reference, we will use the following value as our benchmark:

\[
\text{value of portfolio tomorrow} = (\text{value of portfolio today}) \times e^{r/365}
\]

\[= \$1,119,800(1.000164) = \$1,119,984\]

If you recall, with 29 days to maturity and other things equal, the delta will have decreased to 0.5727. Obviously, this means fewer shares of stock are required to delta hedge the 20,000 call options. The manager will now need only 20,000(0.5727) = 11,454 shares. The manager should sell 24 (11,478 – 11,454) shares and invest the proceeds at the risk-free rate. Thus, the entire value of the original position will continue to grow at the risk-free rate.

Suppose the stock price changes on the next day (e.g., \(S_{T - 29\text{ days}} = \$101\)). We know that the delta will increase from the increase in price but it will tend to fall from the passage of time. From what we have seen so far, it will not be surprising that the price change will dominate and the value of delta will increase to 0.7040 as calculated in the Black-Scholes option pricing model.

**Example: Adjusting a delta hedge**

The initial value of a call is \$1.40, which has a delta of 0.5739 and 30 days to maturity. The price of the stock was \(S_{T - 30\text{ days}} = \$100\). A dealer sells 200 contracts or 20,000 calls. To delta hedge the position, the dealer purchases 11,478 shares of the stock at \$100/share. A day later, the price of the stock is now \(S_{T - 29\text{ days}} = \$101\) and all of the other inputs are the same, which gives \(C = \$1.97\) and \(\text{delta} = 0.7040\). Determine how the manager should adjust the hedge.
Answer:
Recall from the previous example that the original value of the portfolio at T= 30 days was $1,119,800. For a perfectly hedged delta portfolio, at T = 29 days, the value of the portfolio will increase by a factor of $e^{0.06/365}$, where 0.06 / 365 represents a day’s worth of interest. The manager must purchase:

$$\text{number of shares} = (\text{number of calls})(\text{new delta} – \text{old delta})$$

$$= (20,000)(0.7040 – 0.5739) = 2,602 \text{ new shares}$$

To purchase the new shares, the manager will borrow at the risk-free rate. Borrowing and investing the money in the delta-hedged portfolio will not change the value of the net position, which will continue to grow at the risk-free rate.

In summary, the challenge to the delta-hedging manager is the need to continually rebalance to maintain the hedge. Done correctly, the strategy will earn a risk-free rate of return over time for the delta-hedged portfolio.

Professor’s Note: Delta hedging is simple in concept but challenging in the real world. If you are going beyond the exam material, you may have noted the portfolio started at $1,119,800. But a day later, including the call position, it is worth:

$$\text{next-day value of portfolio} = 101(11,478) – 1.97(20,000) = 1,119,878$$

The $78 increase reflects earning the risk-free rate on the hedged position but also the inherent issues in implementing the strategy. We had to use whole shares, so there is some rounding, but the more significant issue is the position must be continually rebalanced. This means on an instantaneous basis, which is impossible. This is the gamma effect we are about to discuss.

We have also just illustrated that perfect hedging is rarely possible. If you did delta hedging for real, you would typically run extensive simulations to quantify how serious the discrepancies can be and help you decide how often to rebalance. Do not expect clear answers; less frequent rebalancing increases the risk, but more frequent rebalancing increases the cost.

**THE GAMMA EFFECT**

**LOS 27.f:** Interpret the gamma of a delta-hedged portfolio and explain how gamma changes as in-the-money and out-of-the-money options move toward expiration.

Gamma measures the change in the value of delta with a change in the value of the underlying stock. Expressing the relationship as an equation:

$$\gamma = \frac{\Delta \text{delta}}{\Delta \text{stock}} = \frac{\text{change in the value of delta}}{\text{change in the price of the underlying}}$$
Professor’s Note: Gamma is called a second-order effect because it is less significant than delta in the analysis and because gamma is the second derivative of the option price relative to the underlying stock price. It is the rate of change in delta.

An important relationship exists between the value of gamma and the risk of the delta-hedged position. If delta were perfectly linear, gamma would equal 0, and one could perfectly delta hedge an option position. The return to the delta hedge would be the risk-free rate.

Think of it this way: if gamma were zero, delta must be constant because it would be the slope of a straight line. That means the price of the option will move by the same amount for a given change in the price of the underlying stock, either negative or positive. The manager would never have to adjust the delta hedge.

Unfortunately, we know that the change in the value of an option is not linear (i.e., the value of delta changes as the price of the underlying changes). Therefore, gamma must be non-zero and there must be risk associated with the delta hedge. In fact, the greater the value of gamma, the more risk in the position (i.e., more variability in the value of the option).

Remember the relationship of delta to the price of the underlying relative to the option’s strike price (i.e., moneyness of the option). If a call option is in-the-money, its delta will generally be above 0.5, and as it approaches expiration, its delta approaches 1.0. Likewise, if the option is out-of-the-money, its delta will usually be below 0.5, and as it approaches expiration, its delta falls to zero.

As an at- or near-the-money option approaches expiration, its delta will tend to move quickly to either one or zero, depending on the direction of the stock price movement. Thus, gamma of an at-the-money option is greatest near the expiration date. When option values are subject to large changes (i.e., when gamma is large), the position faces the most risk, and a delta hedger is more likely to gamma hedge. The hedge entails combining the underlying stock position with two options positions in such a manner that both delta and gamma are equal to zero.

For the Exam: The exact two-option strategy is not discussed in the curriculum, so you will not have to describe it on the exam. Just remember that the risk of a delta hedge is greatest when gamma is large, so delta hedgers will take a position in two options, such that delta and gamma are equal to zero.
KEY CONCEPTS

LOS 27.a
An investor creates a covered call position by buying the underlying security and selling a call option. Covered call writing strategies are used to generate additional portfolio income when the investor believes that the underlying stock price will remain unchanged over the short term.

A protective put (also called portfolio insurance or a hedged portfolio) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, \( P_0 \).

The purchase of the put provides a lower limit to the position at a cost of lowering the possible profit (i.e., the gain is reduced by the cost of the insurance). It is an ideal strategy for an investor who thinks the stock may go down in the near future, yet the investor wants to preserve upside potential.

LOS 27.b
There are many strategies that combine calls, puts, and the underlying asset.
- A bull spread strategy consists of a long call and a short call. The short call has a higher exercise price, and its premium subsidizes the long call. It offers gains if the underlying asset's price goes up, but the upside is limited.
- A bear spread strategy is the opposite side of a bull spread. It offers a limited upside gain if the underlying asset's price declines.
- A butterfly spread consists of two long and two short call positions. It offers a return, with a limited upside if the underlying asset price does not move very much.
- A collar strategy is simply a covered call and protective put combined to limit the down and upside value of the position.
- A long straddle is a long call and long put with the same exercise price. The greater the move in the stock price, the greater the payoff from a straddle.
- A box spread strategy combines a long put and a short put with a long call and a short call to produce a guaranteed return. That return should be the risk-free rate.

LOS 27.c
The basic approach is simple and consists of steps.

To hedge a future borrowing, purchase a call on interest rates for protecting from increasing rates.

To hedge a future lending, purchase a put on interest rates for protection from declining interest rates.

1. Assume that at time 0 the option to hedge the risk is purchased, and the purchase price is financed by borrowing at a rate reflecting the primary loan spread for a net CF of zero at time 0.

2. At time \( t \) when the primary loan occurs, net the cash flow of the primary loan with the option premium financing repayment to determine a net CF at time \( t \).
3. At time $T$ when the primary loan is repaid, net the repayment of primary loan cash flow and any payoff on the option for a net CF at time $T$.

4. Calculate the EAR between $T_T$ and $T_t$ net CFs.

**LOS 27.d**

An interest rate cap is a series of interest rate calls with the same strike rate but different expiration dates. Settlements are at the end of each period but are based on rates at the beginning of each period.

An interest rate floor is a series of interest rate puts with the same strike rate but different expiration dates. As with the cap, settlements are at the end of each period based on rates at the beginning of each period.

An interest rate collar is a combination of cap and floor where the investor is long one and short the other. A short cap and long floor would be of use to a lender of floating-rate loans. The collar will guarantee a range of income for the total position to the lender. A long cap and short floor will guarantee a floating-rate borrower a range of interest costs on the loan.

**LOS 27.e**

Delta hedging generally refers to immunizing the value of an option position from changes in the value of the underlying asset. One such procedure is to immunize a naked call position with a long position in the underlying asset. Correctly done, over time this portfolio should increase in value at the risk-free rate. The problem is that the correct amount of shares to hold for the delta hedge will change. Over time, delta will change and the manager should periodically adjust the asset’s position and invest in or borrow at the risk-free asset to keep the hedge.

Delta is the change in the price of an option for a 1-unit change in the price of the underlying security.

$$\Delta_{\text{call}} = \frac{C_1 - C_0}{S_1 - S_0} = \frac{\Delta C}{\Delta S}$$

where:
- $\Delta_{\text{call}}$ = delta of the call option
- $\Delta C$ = change in the price of the call over a short time interval
- $\Delta S$ = change in the price of the underlying stock over a short time interval

For a long call: $0 \leq \Delta \leq 1$. For long puts and short calls, delta fluctuates between 0 and −1.

**LOS 27.f**

$$\gamma = \frac{\text{change in the value of delta}}{\text{change in the price of the underlying}} = \frac{\Delta \Delta_{\text{delta}}}{\Delta_{\text{stock}}}$$

As an at- or near-the-money option approaches expiration, its delta will tend to move quickly to either one or zero, depending on the direction of the stock price movement. Thus, gamma is highest for at-the-money options near the expiration date. This makes delta hedging riskier.
Study Session 15  
Cross-Reference to CFA Institute Assigned Reading #27 – Risk Management Applications of Option Strategies

**CONCEPT CHECKERS**

1. The holder of a long straddle *most likely* will have a net loss if the asset's price:  
   A. stays the same.  
   B. moves up.  
   C. moves down.

2. Which of the following option combinations cannot be used to construct a butterfly?  
   A. Two put option contracts.  
   B. Four call option contracts.  
   C. Two call options and two put options.

3. A stock trades at 51. Calls with strike prices of 47 and 53 are priced at 5.25 and 0.75, respectively. **Compute** the initial investment for a bull spread and the breakeven price or prices of the spread.

4. The EUR is trading at USD 1.035. A trader expects the EUR to become much more volatile than reflected in current option prices. Puts and calls on the EUR are available. Puts with a strike of USD 0.98 are trading at USD 0.005 and with a strike of USD 1.04 are trading at USD 0.017. Calls with a strike of USD 0.98 are trading at USD 0.068 and with a strike of USD 1.04 are trading at USD 0.004. **Compute** the breakeven price or prices of the correct option strategy.

5. For hedging risk, owning an interest rate put would *most likely* be useful for a:  
   A. variable-rate borrower.  
   B. fixed-rate lender.  
   C. variable-rate lender.

6. A cap contract has a notional principal of $5 million, a strike rate of 5%, and an annual frequency of settlement. If the reference rate is 6% for a given settlement date, what is the payoff to the agent long the cap for that period?  
   A. $25,000.  
   B. $50,000.  
   C. $100,000.

Use the following information to answer Questions 7, 8, and 9. Answer the questions in order.

An option dealer sold call options on 1,667 shares of stock. The underlying stock is priced at $70 per share. The options have a delta of 0.60.

7. How many shares of stock must the dealer buy to hedge his price risk with a delta hedge?  
   A. 669.  
   B. 1,000.  
   C. 2,777.

8. **Level 3 Book 4.indb**

9. **Page 202**
8. If the delta associated with the call option changes from 0.60 to 0.70, what will the dealer do?
   A. Buy shares and borrow funds.
   B. Buy shares and lend funds.
   C. Sell shares and lend funds.

9. If the dealer implemented the required hedge from the first question and rebalanced as required in the second question, the excess profit during the time period between initiation and rebalancing is most likely:
   A. zero.
   B. positive.
   C. negative.

10. In 60 days, a bank plans to lend $10 million for 90 days. The lending rate is LIBOR plus 200 basis points. The current LIBOR is 4%. The bank buys a put that matures in 60 days with a notional principal of $10 million, 90 days in underlying, and a strike rate of 5%. The put premium is $2,000. Calculate the effective annual rate of the loan if at expiration the LIBOR = 4.5%, and if the LIBOR = 6.5%.

11. On August 1, a bank enters a 2-year, zero-cost collar for a $20 million portfolio of floating-rate loans by buying the floor and selling the cap. The floor strike is 2.5%, the cap strike is 4.7%, and the reference rate is LIBOR. The interest payments on the loan assets are LIBOR plus 240 basis points. The collar's semiannual settlement dates exactly match the dates when the floating-rate payments are made: August 1 and February 1 over the next two years. Payments made August 1 cover 181 days and payments made February 1 cover 184 days. Current LIBOR is 4.1%. The values of LIBOR on the next three settlement dates are 2.4%, 5%, and 5%. Calculate the actual interest rate payments (to the bank), settlements, and effective interest payments.

For more questions related to this topic review, log in to your Schweser online account and launch SchweserPro™ QBank; and for video instruction covering each LOS in this topic review, log in to your Schweser online account and launch the OnDemand video lectures, if you have purchased these products.
ANSWERS – CONCEPT CHECKERS

1. A  If the asset’s price is between the put and call breakeven points, the long straddle produces a net loss.

2. A  By examining the payoff pattern of a butterfly, it is clear that there must be three strike prices used. With only two option contracts, there can only be two strike prices. Either of the two other combinations are possible. Starting from the left side of the payoff pattern: a long call with a lower strike price, plus two short calls with a medium strike price, plus a long call with a higher strike price will work. Alternatively, beginning from the middle of the payoff pattern, and utilizing a reverse straddle (sell a medium strike price call and put) plus buy a put with a lower strike price and buy a call with a higher strike price will achieve the same payoff pattern. Be prepared to understand combinations such as this that were not specifically discussed in the reading. It is just combining the four intrinsic value patterns.

3. Buy the 47 call at 5.25 and sell the 53 call at 0.75 for an initial investment of 4.50.

BE can be computed from either max loss or max gain. Looking at the graph for a bull spread the max loss is at $S = 47$. Both calls are worthless. The loss is the initial investment of 4.50. Again looking at the graph, the stock must increase 4.50 to 51.50 for BE.

4. Buy ATM puts and calls on the EUR. The 1.04 strike price is the closest to ATM. Buying the call and put will cost: 0.004 + 0.017 = 0.021. Looking at the graph for a straddle, this is the max loss and occurs if the EUR closes at 1.04. For BEs, the EUR must decrease or increase 0.021 to USD 1.019 or 1.061.

5. C  The put pays the floor holder when interest rates fall, so they would hedge the risk of a variable rate.

6. B  payoff = $50,000 = ($5,000,000)(0.06 – 0.05)(1)$

7. B  The short call position will lose value if the underlying increases. The loss is 0.6 for each $1 increase in the stock price. To hedge, the dealer will buy $1,667 \times 0.6 = 1,000$ shares.

8. A  The hedge is now $1,667 \times 0.7 = 1,167$ shares. The purchase of another 167 shares is financed by borrowing.

9. C  Examine the graph of a short call position. As the underlying increases, the loss on the short call accelerates (the delta increases). Any delay in rebalancing the hedge results in the loss on the short calls exceeding the gain on the long shares as the underlying increases.

10. First, we compute the effective amount the bank parts with or lends at time of the loan. This means computing the future value of the premium:

    future value of premium = $2,020 = $2,000[1 + (0.04 + 0.02)(60 / 360)]

Thus, the cash outflow at the loan’s inception is $10,002,020. At the given LIBOR rates of 4.5% and 6.5%, the put’s payoffs are:

    LIBOR = 4.5%: payoff = $12,500 = $10,000,000[max(0, 0.050 – 0.045)(90 / 360)]
    LIBOR = 6.5%: payoff = $0 = $10,000,000[max(0, 0.050 – 0.065)(90 / 360)]
The interest income earned is:

\[
\text{LIBOR} = 4.5\%: \quad \text{int. income} = \$162,500 = \$10,000,000 \times (0.045 + 0.020) \left(\frac{90}{360}\right)
\]
\[
\text{LIBOR} = 6.5\%: \quad \text{int. income} = \$212,500 = \$10,000,000 \times (0.065 + 0.020) \left(\frac{90}{360}\right)
\]

The effective rate earned is:

\[
\text{LIBOR} = 4.5\%:\quad \text{EAR} = \left[\frac{\$10,000,000 + \$162,500 + \$12,500}{\$10,002,020}\right]^{\left(\frac{365}{90}\right)} - 1
\]
\[
\text{LIBOR} = 4.5\%:\quad \text{EAR} = \left[\frac{\$10,175,000}{\$10,002,020}\right]^{\left(\frac{365}{90}\right)} - 1
\]
\[
\text{EAR} = 0.0720 = 7.2\%
\]

\[
\text{LIBOR} = 6.5\%:\quad \text{EAR} = \left[\frac{\$10,000,000 + \$212,500 + \$0}{\$10,002,020}\right]^{\left(\frac{365}{90}\right)} - 1
\]
\[
\text{LIBOR} = 6.5\%:\quad \text{EAR} = \left[\frac{\$10,212,500}{\$10,002,020}\right]^{\left(\frac{365}{90}\right)} - 1
\]
\[
\text{EAR} = 0.0881 = 8.81\%
\]

11. The first collar expiration will occur on February 1 for payment on August 1. There will be no collar payment on the first February 1. The payoffs on the derivatives are:

Floorlets:

Year 1

payoff on Feb. 1 = N/A
payoff on Aug. 1 = \$10,056 = \$20,000,000[\max(0, 0.025 – 0.024)(181 / 360)]

Year 2

payoff on Feb. 1 = \$0 = \$20,000,000[\max(0, 0.025 – 0.050)(184 / 360)]
payoff on Aug. 1 = \$0 = \$20,000,000[\max(0, 0.025 – 0.050)(181 / 360)]

Caplets:

Year 1

payoff on Feb. 1 = N/A
payoff on Aug. 1 = \$0 = \$20,000,000[\max(0, 0.024 – 0.047)(181 / 360)]

Year 2

payoff on Feb. 1 = \$30,667 = \$20,000,000[\max(0, 0.050 – 0.047)(184 / 360)]
payoff on Aug. 1 = \$30,167 = \$20,000,000[\max(0, 0.050 – 0.047)(181 / 360)]

The interest payments are:

pmt. on Feb. 1 = \$664,444 = \$20,000,000(0.041 + 0.024)(184 / 360)
pmt. on Aug. 1 = \$482,667 = \$20,000,000(0.024 + 0.024)(181 / 360)
pmt. on Feb. 1 = \$756,444 = \$20,000,000(0.050 + 0.024)(184 / 360)
pmt. on Aug. 1 = \$744,111 = \$20,000,000(0.050 + 0.024)(181 / 360)
The following table illustrates how the payments and payoffs combine to give an effective rate for each period.

<table>
<thead>
<tr>
<th>Settlement</th>
<th>Year</th>
<th>Actual Interest</th>
<th>Floor Payoffs</th>
<th>Cap Payoffs</th>
<th>Effective Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb. 1</td>
<td>1</td>
<td>$664,444</td>
<td>N/A</td>
<td>N/A</td>
<td>$664,444</td>
</tr>
<tr>
<td>Aug. 1</td>
<td>1</td>
<td>$482,667</td>
<td>$10,056</td>
<td>0</td>
<td>$492,723</td>
</tr>
<tr>
<td>Feb. 1</td>
<td>2</td>
<td>$756,444</td>
<td>0</td>
<td>−$30,667</td>
<td>$725,777</td>
</tr>
<tr>
<td>Aug. 1</td>
<td>2</td>
<td>$744,111</td>
<td>0</td>
<td>−$30,167</td>
<td>$713,944</td>
</tr>
</tbody>
</table>
The following is a review of the Risk Management Applications of Derivatives principles designed to address the learning outcome statements set forth by CFA Institute. Cross-Reference to CFA Institute Assigned Reading #28.

**RISK MANAGEMENT APPLICATIONS OF SWAP STRATEGIES**

**Exam Focus**

Swaps are commonly used to modify risk in portfolios and on balance sheets. Virtually any swap analysis starts with the swap diagram. The swap diagram begins with the initial situation of the principal involved. Then an appropriate swap can be designed to accomplish the desired objective. From the diagram, the duration of the swap can be inferred. Be prepared for both conceptual as well as calculation questions.

**Using Swaps to Convert Loans From Fixed (Floating) to Floating (Fixed)**

**LOS 28.a:** Demonstrate how an interest rate swap can be used to convert a floating-rate (fixed-rate) loan to a fixed-rate (floating-rate) loan.

The most common interest rate swap is the *plain vanilla* interest rate swap. In this swap, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Payments are in the same currency, so only the net payment is exchanged.

Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, because the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called *notional principal*.

**Example: Converting fixed to floating and floating to fixed**

Company X has a $100 million, 2 year, 5% fixed rate semi-annual pay debt. Payments are actual day count over a 360-day year. The company expects interest rates to fall and would prefer to have a floating rate debt.

Company Y has a $100 million, two-year, floating rate semiannual pay debt at LIBOR plus 100 basis points. Payments are actual day count over a 360-day year. The company expects interest rates to rise and would like to use a swap to convert the debt to fixed rate.

A $100 million, two-year, 5.5% semiannual pay swap versus LIBOR plus 125 basis points is available to both X and Y.
The net result is Company X is now paying a synthetic floating rate of LIBOR plus 75 basis points.

- Pay 5% on fixed rate debt.
- Pay LIBOR plus 1.25% on swap.
- Receive 5.5% on swap.

Company X has effectively removed its fixed-rate liability exposure and converted it to LIBOR exposure. Its fixed-rate debt would have had a higher duration and now its debt has the lower duration of a floating rate liability. Company X is effectively speculating LIBOR will fall.

The net result is Company Y is now paying a synthetic fixed rate of 5.25%.

- Pay LIBOR plus 1% on floating rate debt.
- Receive LIBOR plus 1.25% on swap.
- Pay 5.5% on swap.

Company Y has effectively removed its floating rate liability exposure and converted it to fixed rate exposure. Its floating rate debt would have had a lower duration and now its debt has the higher duration of a fixed rate liability. Company Y is effectively speculating LIBOR will rise.
**Duration of an Interest Rate Swap**

**LOS 28.b: Calculate and interpret the duration of an interest rate swap.**

The duration properties of swaps are another reason for their popularity. Each counterparty in a swap is essentially either of the following:

- Long a fixed cash flow and short a floating cash flow.
- Short a fixed cash flow and long a floating cash flow.

You should recall that duration is the sensitivity of an asset’s price to changes in a relevant interest rate. Here are two important points with respect to fixed and floating-rate instruments:

1. For fixed-rate instruments, duration will be higher because the change in interest rates will change the present value of the fixed cash flows.
2. For floating-rate instruments, duration is close to zero because the future cash flows vary with interest rates, and the present value is fairly stable with respect to changes in interest rates.

A floating-rate instrument can have a non-zero duration if its next cash flow has been set, which is the case with swaps. The convention is to treat the duration of the floating rate side of the swap as being half the reset period. For example, for a 6-month reset, the duration would be taken to be 0.25, for a quarterly reset it would be taken to be 0.125, etc.

Because we know that the duration of a zero-coupon bond is its maturity, the duration of the floating payments where the next payment is known will be the time to the next payment. At inception or just after a settlement for a quarterly reset swap, the duration of the floating payments is 0.25; for a semiannual reset swap, the duration is 0.5; et cetera. Just before the payment is due, however, the duration is 0.0. Hence, the average duration of a floating instrument over the reset period is one-half the length of its settlement periods.

For a pay-floating counterparty in a swap, the duration can be expressed as:

\[ D_{\text{pay floating}} = D_{\text{fixed}} - D_{\text{floating}} > 0 \]

Because the floating-rate payor receives fixed cash flows, taking the receive-fixed/pay-floating position in a swap increases the dollar duration of a fixed-income portfolio. The modified duration of the portfolio will move an amount determined by (1) the relative values of the notional principal of the swap and the portfolio’s value and (2) relative values of the modified duration of the swap and that of the portfolio.
Professor’s Note: Calculating swap duration is simple; it is the difference in the durations of the two sides of the swap. The swap diagram is an easy way to remember how to calculate swap duration. The arrow coming in represents an asset; add the duration of what is received on the swap. The arrow going out represents a liability; subtract the duration of what is paid on the swap. The result is the swap duration.

Example: Pay-floating swap duration

At the inception of a 2-year swap, the duration of the fixed payments is 1.1, and the duration of the floating payments is 0.25. What is the duration of the swap to the pay-floating party to the swap?

Answer:

The duration of the swap is $1.1 - 0.25 = 0.85$.

This is +0.85 to the receive fixed and pay floating counterparty.

**Market Value Risk and Cash Flow Risk**

**LOS 28.c:** Explain the effect of an interest rate swap on an entity’s cash flow risk.

It is common to refer to converting a floating-rate liability to fixed-rate as a hedge. In the sense that it reduces cash flow risk, it is a hedge. However, it is essentially converting highly visible cash flow risk into less visible market value risk.

Cash flow risk is reduced by entering the swap because the uncertain future floating-rate payments on the liability are essentially converted to fixed payments that can be more easily planned for and budgeted, resulting in a reduction in cash flow risk. However, the low duration of the floating-rate liability is now converted to the higher duration of a fixed-rate liability. The liability market value will now fluctuate more as interest rates change. For example, if interest rates fall, the liability will rise in market value, creating a corresponding decline in the firm’s theoretical market value of equity.

Some argue that these changes in market value are unrealized, which is true. They are nonetheless real, and financial theory would clearly suggest they should affect the market value of the firm and of the equity. The swap reduces cash flow risk but increases market value risk.
**Using Swaps to Change Duration**

**LOS 28.d:** Determine the notional principal value needed on an interest rate swap to achieve a desired level of duration in a fixed-income portfolio.

The duration of the portfolio plus a swap position (i.e., the target duration) is calculated as:

\[ V_p (MD_T) = V_p (MD_p) + NP (MD_{swap}) \]

where:
- \( V_p \) = original value of the portfolio
- \( MD_i \) = modified duration \( i \) (\( i = \) swap, target, portfolio without swap)
- \( NP \) = notional principal of the swap

Usually, the portfolio manager selects a swap of a certain maturity which determines the modified duration of the swap, \( MD_{swap} \). He then selects the \( NP \) that will achieve the desired \( MD_T \). Rearranging, we can solve for the amount of notional principal necessary to achieve the target duration:

\[ NP = \left( \frac{V_p (MD_T - MD_p)}{MD_{swap}} \right) \]

**Professor’s Note:** This is just a variation of the basic bond hedging formula but set up to calculate notional principal rather than the number of contracts.

**Example: Determining the notional principal**

A manager of a $60 million fixed-income portfolio with a duration of 5.2 wants to lower the duration to 4.0. The manager chooses a swap with a net duration of 3.1. What \( NP \) should the manager choose for the swap to achieve the target duration?
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #28 – Risk Management Applications of Swap Strategies

Answer:

From the given information, we have:

\[ V_p = 60,000,000 \]
\[ MD_p = 5.2 \]
\[ MD_{swap} = 3.1 \]
\[ MD_T = 4.0 \]
\[ NP = V_p \left( \frac{MD_T - MD_p}{MD_{swap}} \right) \]
\[ NP = 60,000,000 \left( \frac{4.0 - 5.2}{3.1} \right) = 23,225,806 \]

Because the manager wants to reduce the duration of his portfolio, he should take a receive-floating/pay-fixed position in the swap with that notional principal. Remember that a receive-floating swap has a negative duration, so we enter –3.1 in the equation.

For the Exam: Be sure to enter the net duration of the swap correctly in the denominator of the equation (i.e., negative if pay-fixed; positive if receive-fixed). You can tell if you have entered it correctly because the sign on the notional principal should always be positive.

Warm-Up: Currency Swaps

A currency swap is different from an interest rate swap in two very important ways:

1. There are two notional principals, one in each currency, and the counterparties generally exchange the principals on the effective date and return them at the maturity date.

2. Because the cash flows in a currency swap are denominated in different currencies, the periodic interest payments are not usually settled on a net basis, so each counterparty makes a payment to the other in the appropriate currency.

A plain vanilla currency swap is one in which the floating-rate cash flows (usually based on LIBOR) are in dollars, while the other cash flows (in another currency, like euros) are based on a fixed rate. However, because swaps are OTC instruments, the counterparties can design them any way they choose (e.g., floating for floating, dollar floating and foreign fixed, fixed for fixed).

One of the more common reasons for a firm to engage in a currency swap is to gain access to loanable funds in a foreign currency that might be too costly to obtain from a bank, the reason being that the firm does not have close relationships with banks in the country of the desired currency.
A firm may also have issued a foreign-currency bond earlier, and now the firm wishes to convert it into a domestic obligation. A swap can help with that, too. If a U.S. company has a fixed-rate note denominated in euros and wishes to make it a synthetic dollar loan, the U.S. firm can enter into a receive-euro/pay-dollar swap. Because the plain vanilla currency swap exchanges fixed foreign currency for floating dollars, the U.S. firm’s synthetic position will now be a floating-rate dollar obligation.

The following demonstration illustrates the mechanics of the swap in combination with the loans on both sides of the swap. Also, for added measure, we put the dealer in the mix, too! Dealers are involved in most transactions, and you may see them as part of an exam question.

**Example: Currency swap**

A U.S. company has a liability of €10 million in fixed-rate bonds outstanding at 6%. A German company has a $15 million FRN outstanding at LIBOR. The exchange rate is $1.5/€. The U.S. company enters into a plain vanilla currency swap with the swap dealer in which it pays LIBOR on $15 million and receives the swap rate of 6.0% on €10 million. The German company also enters into a plain vanilla currency swap with the same dealer, in which it pays a swap rate of 6.1% on €10 million and receives LIBOR on $15 million. One-year LIBOR is currently 5.2%.

**Calculate** each party’s net borrowing cost, the principal cash flows at the initiation and maturity of the contract, and first-year cash flows (assume annual settlement).

**Answer:**

The cash flow for each settlement date for this plain vanilla currency swap is illustrated in the figure below.

**Cash Flows for a Plain Vanilla Currency Swap**

The U.S. company’s net borrowing cost: LIBOR on $15 million

\[(6\% \text{ on } €10 \text{ million}) + (\text{LIBOR on } $15 \text{ million} - 6\% \text{ on } €10 \text{ million})\]

The German company’s net borrowing cost: 6.1% on €10 million

\[(\text{LIBOR on } $15 \text{ million}) + (6.1\% \text{ on } €10 \text{ million} - \text{LIBOR on } $15 \text{ million})\]
Study Session 15
Cross-Reference to CFA Institute Assigned Reading #28 – Risk Management Applications of Swap Strategies

The swap dealer’s spread: 0.1% on €10 million = €10,000

\[(\text{LIBOR on $15 million} - 6\% \text{ on €10 million}) + (6.1\% \text{ on €10 million} - \text{LIBOR on $15 million})\]

Not only are the firms paying in different currencies, but they get access to the funds because they exchange notional principals at the beginning of the swap.

The cash flows of the notional principals at the initiation of the swap are shown in the figure below.

Exchange of Notional Principals

The U.S. Company

At the end of the first year, the U.S. company pays interest on its euro borrowing. It pays LIBOR and receives euros under the swap (the negative sign means outflow):

- interest on euro borrowing = –€600,000 = €10,000,000 × 0.060
- euros received under swap = €600,000 = €10,000,000 × 0.060
- U.S. dollars paid under swap = –$780,000 = $15,000,000 × 0.052
- net cash flow = –$780,000

At the beginning of the period, the U.S. company gets a dollar principal and will pay dollars on the amount that was once a euro loan.

The German Company

The German company gets euros and will pay interest on its U.S. dollar borrowing. It receives LIBOR and pays euros under the swap:

- interest on U.S. dollar borrowing = –$780,000 = $15,000,000 × 0.052
- euros paid under swap = –€610,000 = €10,000,000 × 0.061
- U.S. dollars received under swap = $780,000 = $15,000,000 × 0.052
- net cash flow = –€610,000

The Swap Dealer

The net cash flow to the swap dealer is:

- euros received from German firm = €610,000 = €10,000,000 × 0.061
- euros paid to U.S. firm = €600,000 = €10,000,000 × 0.060
- net cash flow = €10,000 = €10,000,000 × 0.001
The principal cash flows at maturity of the swap are shown below.

Cash Flows at the Maturity of the Swap

LOS 28.e: Explain how a company can generate savings by issuing a loan or bond in its own currency and using a currency swap to convert the obligation into another currency.

As mentioned earlier, a counterparty may use a currency swap to gain access to a foreign currency at a lower cost. Borrowing in a foreign country via a foreign bank may be difficult, and the interest rates may be high. A U.S. firm that wishes to initiate a project in a foreign country, say Korea, might not have the contacts necessary to borrow Korean currency (the won) cheaply. It may have to pay a high interest rate, such as 9%. A Korean counterparty may exist that would like to borrow dollars to invest in the United States but finds that banks in the United States charge 7.2% because they are unfamiliar with the Korean firm.

The U.S. firm can borrow at 6% in the United States because it has established relationships with those banks. It swaps the principal (borrowed dollars) with the Korean counterparty for the won, which the Korean firm borrowed at 7% in Korea. The U.S. firm uses its proceeds from its new business to pay the won interest to the Korean counterparty, who in turn pays won interest on its bank loan to the Korean bank. The Korean firm pays the dollar interest to the U.S. firm, who in turn pays dollar interest on its loan to the U.S. bank.

Here are the important points to this exchange:

- The U.S. firm is now paying 7% on a won loan on which it would have had to pay 9% if it had borrowed from a Korean bank.
- The Korean firm is now paying 6% on a dollar loan on which it would have had to pay 7.2% if it had borrowed from a U.S. bank.
An easier way to understand the analysis is to draw the swap diagram:

**Figure 3: Effect of Currency Swap Cash Flows on Interest Payments**

A dealer might have increased the swap interest rates 10 basis points for each counterparty with the dealer earning the spread. But the resulting 7.1% for the U.S. firm is still less than 9%, and the resulting 6.1% is still less than 7.2% for the Korean firm.

**Converting Foreign Cash Receipts**

**LOS 28.f: Demonstrate how a firm can use a currency swap to convert a series of foreign cash receipts into domestic cash receipts.**

Dealers will contract with a firm in a currency swap that does not require an exchange of notional principals. This essentially becomes a series of exchange-rate purchases in the future at a fixed exchange rate. The amounts exchanged are a function of both the current exchange rate and interest rates (swap rates) in the countries involved.

As an example, let’s consider a U.S. firm that wishes to convert its quarterly cash flows of €6 million each to dollars upon receipt. The exchange rate is currently €0.8/$, and the swap rates in the United States and Europe are 4.8% and 5%, respectively. To obtain the swapped dollar cash flow, we first back out the notional principal in euros, translate this to a dollar notional principal, and then calculate the interest in dollars:

\[
\text{NP} = \frac{6,000,000}{0.05/4} = 480,000,000
\]

The corresponding dollar amount is \(\frac{480,000,000}{€0.8/$} = $600,000,000\). The quarterly interest payments on this amount would be \(600,000,000(0.048 / 4) = $7,200,000\).

The swap would then allow the firm to exchange its €6,000,000 quarterly inflow for $7,200,000 per period. The maturity of the swap would be negotiated to meet the needs of the firm. You should note that no exchange of principals was required.
For the Exam: Follow these steps in determining the appropriate swap:

1. Divide the foreign cash flow received by the foreign interest rate to determine the corresponding foreign-denominated notional principal (NP).
   a. This is the foreign NP that would have produced the foreign cash flow at the given foreign interest rate.

2. Using the current exchange rate, convert the foreign NP into the corresponding domestic NP.

3. Enter a swap with this NP.
   a. Pay the foreign cash flows received on the assets and receive the equivalent domestic amount.
   b. The amount of each domestic cash flow is determined by multiplying the domestic interest rate by the domestic NP.

Example: Currency swap without a notional principal exchange

A firm will be receiving a semiannual cash flow of €10 million. The swap rates in the United States and Europe are 6% and 5%, respectively. The current exchange rate is €0.9/$. Identify the appropriate swap needed to convert the periodic euro cash flows to dollars.

Answer:

For the euros, the NP = €10,000,000 / (0.05 / 2) = €400,000,000. The corresponding dollar amount is €400,000,000 / 0.9 = $444,444,444. Using these values for the swap, the firm will give the swap dealer €10,000,000 every six months over the maturity of the swap for:

$444,444,444(0.06 / 2) = $13,333,333

EQUITY SWAPS

LOS 28.g: Explain how equity swaps can be used to diversify a concentrated equity portfolio, provide international diversification to a domestic portfolio, and alter portfolio allocations to stocks and bonds.

An equity swap is a contract where at least one counterparty makes payments based upon an equity position. The other counterparty may make payments based upon another equity position, a bond, or just fixed payments. We will begin with that example.
Example: Equity swap with fixed payments

A firm owns a stock portfolio that is closely correlated with the S&P 500. The firm is concerned that the stock market will fall over the next year. A 1-year, quarterly equity swap is available with a notional principal equal to the value of the portfolio and a fixed rate of 7%. Diagram the net quarterly cash flows to a hedge.

Answer:

The net effect for the firm is a fixed-rate return of $7 \%/4 = 1.75\%$ per period as shown below.

![Diagram of Quarterly Cash Flows to an Equity Swap]

The swap does create some secondary risks, including:

- Counterparty risk if the swap dealer experiences difficulty and is unable to make the swap payments as expected to the firm.
- Basis risk if the return on the stock portfolio does not exactly match the S&P return payments the firm must make on the swap.
- Some cash flow risk in that the S&P payments to the swap dealer are unlikely to match the fixed payments to the firm. If a net is paid, the firm must have the cash available. Note: if the firm had sold the stock and actually bought fixed rate bonds, there is no cash flow risk as the firm just collects coupons.

Swaps to Create International Diversification

As a variation on the previous example, suppose the firm had a $500 million equity portfolio and swapped $100 million to receive the 7% fixed rate. The firm effectively has 20% fixed rate bond exposure. Now the firm does a second swap for $50 million, paying the S&P return and receiving the return for the MSCI, an international stock index. This would further diversify the portfolio by creating a 10% allocation to international equity.

This second swap will have the same secondary risks as the first swap, but the cash flow risk is potentially even greater because the firm is receiving an index return rather than a known fixed rate. Consider the worst case scenario, where the S&P return paid is very large due to a high return on the S&P but the MSCI return is a large negative. The firm contracted to receive the MSCI return and when that return is negative, that means the firm must pay the MSCI return. The firm could end up making two payments with no receipt in a worst case scenario. Notice paying the MSCI return out when it is negative.
simply replicates the loss in value that would have occurred if the firm owned the MSCI and it declined. However, the swap requires cash to be paid, while owning the MSCI would only be a decline in market value. Economically they are the same result but the swap creates cash flow issues.

Benefits of using the swap are that transaction costs for the swap are generally lower than actually selling domestic stocks to buy international stocks. The swap can be for a defined period if this is a temporary exposure that is desired. Also the swap can be structured as payment in U.S. dollars to limit the foreign currency exposure from owning foreign denominated assets and any need to hedge the currency exposure. The dealer may consider these factors in pricing the swap, so the firm should consider whether the pricing is still attractive (to the firm).

Example: Diversifying concentrated positions

An investor has an overweighed 30,000 share position in a stock with a current market value of $80 per share and a dividend yield of 2%. The investor wishes to reduce the position by half for a position in the S&P 500. Demonstrate how this can be accomplished with a swap.

Answer:

The owner of the stock would probably approach a dealer and swap the returns on $1.2 million = (30,000 × $80) / 2 worth of the stock for the returns on a $1.2 million investment in the S&P 500. Each settlement period (e.g., quarter) the total return on each position is calculated. The net amount is transferred between the parties.

Changing Allocations of Stock and Bonds

Another type of swapping of index returns can occur between, for example, large- and small-cap stocks. A firm with an equity portfolio that is 60% in large-cap stocks, 30% in mid-cap stocks, and 10% small-cap stocks can use a swap to synthetically adjust this position. If the value of the portfolio is $200 million and the firm decides to make the large- and mid-cap exposure equal without touching the small-cap position, then it can become a counterparty in a swap that receives the return of the S&P Mid-Cap 400 Index and pays the return on the Dow Jones Industrial Average Index on a notional principal of 15% of $200 million (i.e., $30 million). Ignoring tracking error, this will synthetically make the portfolio a 45% large-cap, 45% mid-cap, and 10% small-cap stock portfolio over the life of the swap. The small-cap position is unaffected.

This concept can also be applied to the synthetic adjustment of a bond portfolio. A firm with a given portfolio of high-grade and low-grade bonds can enter into a swap that pays the return on an index of one type (e.g., the high-grade) and receives the return on the index of another type (e.g., the low-grade). Do not confuse this with an interest rate swap! In the swap based on bond returns, there is an interest component and a capital gain component just as there is in an equity swap.
Example: Changing allocations of stocks and bonds

We will consider a manager of a $120 million bond portfolio that consists of $80 million in investment-grade corporate bonds and $40 million in U.S. Treasuries. The manager wants to switch the weights. **Demonstrate** how this can be accomplished with a swap.

**Answer:**

Once again, the manager approaches a dealer about swapping the returns on indices like the Barclays Capital Long-Term Treasury Bond Index and the Merrill Lynch Corporate Bond Index. The notional principal will be $40 million.

**INTEREST RATE SWAPTIONS**

**LOS 28.h:** Demonstrate the use of an interest rate swaption 1) to change the payment pattern of an anticipated future loan and 2) to terminate a swap.

**For the Exam:** Be able to explain why and how a manager would use a swaption as well as calculate the payoff or cash flows to the swaption if exercised.

An **interest rate swaption** is an option on a swap where one counterparty (buyer) has paid a premium to the other counterparty (seller) for an option to choose whether the swap will actually go into effect on some future date. The terms of the swap are usually determined at the time of the swaption’s inception, prior to the effective date of the swap. Swaptions can be either American or European in the same way as options. European-style swaptions may only be exercised on the expiration date, whereas an American-style swaption may be exercised on any day up to and including the expiration date.

There are two types of swaptions:

1. **Payer swaption:** A payer swaption gives the buyer the right to be the fixed-rate payer (and floating-rate receiver) in a prespecified swap at a prespecified date. The payer swaption is almost like a protective put in that it allows the holder to pay a set fixed rate, even if rates have increased.

2. **Receiver swaption:** A receiver swaption gives the buyer the right to be the fixed-rate receiver (and floating-rate payer) at some future date. The receiver swaption is the reverse of the payer swaption. In this case, the holder must expect rates to fall, and the swap ensures receipt of a higher fixed rate while paying a lower floating rate.

The key point is that the terms of the underlying swap and the swap fixed rate (SFR) are negotiated and set at the purchase of the swaption. The purchaser of the swaption can wait and see if subsequent market moves make that SFR attractive or not and then decide whether to turn on the underlying swap.
Payer Swaption

• If market interest rates move high enough at the expiration of the swaption such that the new SFR is above the swaption SFR, the holder of the payer swaption will exercise the option. Paying the swaption SFR is better than the terms on a new swap. The swaption swap has positive value.

• If interest rates move low enough such that the new SFR is below the swaption SFR, the holder would let the swaption expire worthless and only lose the premium paid. The swaption swap has negative value.

Receiver Swaption

• If interest rates move high enough and the new SFR is above the swaption SFR, the holder of the swaption would let it expire worthless and only lose the premium paid. The SFR that would be received on the swaption SFR is unattractive and the swaption swap has negative value. A better SFR is available on a new swap.

• If market interest rates move low enough and the new SFR is below the swaption SFR, the swaption will be exercised. The swaption swap has positive value with an above market SFR to be received.

Using Swaptions to Hedge a Future Loan Transaction

A corporate manager may wish to purchase a fixed-rate payer swaption to synthetically lock in a maximum fixed rate to be paid on an FRN to be issued in the future. If interest rates decline, the manager can always let the option expire worthless and take advantage of lower rates. The time line is illustrated in Figure 1. Today the manager enters into a swaption by paying a premium. The option expires at the time the loan will be taken out. For generality, Figure 1 does not specify a floating- or fixed-rate loan.

• The payer swaption would convert a future floating-rate loan to a fixed-rate loan.
• The receiver swaption would convert a future fixed-rate loan to a floating-rate loan.

Figure 4: Swaption and Future Loan

As an example, if a manager is planning to take out a 3-year loan of $10 million at a floating rate, say LIBOR plus 250 basis points, then the manager could hedge the risk of rising interest rates by purchasing a payer swaption with a notional principal of $10 million. (The premium might be $200,000, but the amount is not important for our discussion here. You should just know that an up-front premium is usually required.) The swap would be to receive 90-day LIBOR each quarter, to hedge the loan payments, and pay a fixed rate. The fixed rate might be 3.6% or 0.9% each quarter. At the exercise date of the swaption and the beginning of the loan, one of the following two scenarios will result.
1. The fixed rate on 3-year swaps that pay LIBOR is greater than 3.6%. Then the manager will exercise the swaption to pay the contracted 3.6% and receive LIBOR. We will recall our formula for a floating-rate borrower who is a floating-rate receiver in a swap:

\[
\text{net payment} = NP[\text{swap rate} + \text{(loan spread)}](\frac{D_t}{360})
\]

In this case, the floating-rate loan plus swap will become a synthetic fixed-rate loan with the following quarterly payments (assuming 90-day settlement periods):

\[
\text{net payment} = 10,000,000(0.036 + 0.025)(90 / 360) = 152,500
\]

2. The fixed rate on 3-year swaps that pay LIBOR is less than 3.6%, say 3.2%. Then the manager will let the swaption expire, which means there was no realized benefit for the $200,000 premium paid. The manager may contract, at a zero cost, to enter into a 3-year swap with a fixed rate of 3.2%. In this case, the floating-rate loan plus swap will become a synthetic fixed-rate loan with quarterly payments (assuming 90-day settlement periods):

\[
142,500 = 10,000,000 \times (0.032 + 0.025) \times (90 / 360)
\]

Scenario 2, the no-swaption exercise, could have had the manager not engage in any swap, even at the lower strike rate of 3.2%. That would be up to the manager. The concept is fairly simple. If exercised, the swap does its job. If not exercised, the manager is free to hedge or not hedge.

**Using Swaptions to Potentially Terminate an Existing Swap Early**

The actions outlined previously can easily be modified to apply to other situations. A manager who is under contract in an existing swap can enter into a swaption with the exact characteristics of the existing swap but take the other counterparty’s position. It is possible to match the payments and characteristics because the premium can be adjusted to make the contract worthwhile to the dealer.

Figure 2 has the same general form as Figure 1, but it has been relabeled to depict a cancellation of an existing swap with a swaption.
A manager in a pay-floating swap with a given NP and swap fixed rate (SFR) would simply contract to be the receive-floating counterparty in a swaption that has an exercise date in the future. If the NP and SFR are the same for both the swaption and the swap, then upon exercise the swaption’s cash flows will effectively cancel the cash flows of the existing swap. If the manager buys the swaption from the same dealer with whom the original swap was contracted, the position would effectively be closed. The purchaser of the swaption can then wait until expiration of the swaption and decide if market conditions make it attractive to turn on the swaption swap and effectively cancel the existing swap.

**Synthetically Adding or Removing a Call Feature on Existing Debt**

*Professor’s Note: There is no direct Learning Outcome Statement for the section on bond calls but it is in the assigned text. It would be an unlikely question.*

A company has a 10-year, non-callable bond liability and wishes it were callable in three years. The company would buy a 3-year swaption on a 7-year swap to pay floating and receive fixed. Now assume three years have passed:

- If interest rates are low enough, the company can exercise the swaption and enter the 7-year swap (the remaining term of the bond issue) to receive fixed and pay floating. The fixed receipts on the swap cover the fixed coupon payments on the bond and effectively convert it to floating (at now-low rates). This replicates the economic benefit the company would have received if the bond could have been called. If desired the company could even enter a new additional swap to receive floating (covering the floating payments the company must make on the executed swaption) and pay fixed. Interest rates are down and this new swap SFR will be low. It is as if the company called its debt and refinanced at new lower interest rates.

- If interest rates are high enough that the company would not have wanted to call the bond, the company does nothing and lets the swaption expire worthless.

Another company has a 5-year bond issue outstanding that is callable in 1 year. Further suppose the company does not expect rates to be low enough to make calling the bond worthwhile, or alternatively, the company needs cash today. The company could sell a 1-year swaption on a 4-year swap to pay fixed. Now assume one year has passed:

- If rates are high enough to make the swaption worthless, the purchaser of the swaption (who would receive fixed) will let it expire worthless. With high rates, the company will not call the bond.

- Alternatively if rates fall low enough, the purchaser of the swaption will exercise and receive the swaption’s SFR. In addition, the company will call the bonds in the low rate environment. The net is the company benefits from calling the bond but loses when the swaption is exercised and requires the company to pay fixed. The company gains and loses the benefit of calling the bond and economically is in a position as if the bond had not been callable.
LOS 28.a
Interest rate swaps are used to change the nature of the cash flows (either fixed or floating) on assets and liabilities. A floating-rate (fixed-rate) payment on a liability can be effectively converted to a fixed-rate (floating-rate) by entering a pay-fixed, receive-floating (pay-floating, receive-fixed) swap. The goal is for the cash flow received on the swap to offset the original payment on the liability, such that the nature of the net payment on the liability is opposite from the original. For a floating- (fixed-) rate asset, the manager will enter a pay-floating, receive-fixed (pay-fixed, receive-floating) swap. The goal is to have the payment on the swap offset the receipt on the asset, such that the net receipt is opposite in nature from the original.

LOS 28.b
\[ D_{\text{swap}} = D_{\text{asset}} - D_{\text{liability}} \]

For a pay-floating counterparty in a swap, the duration can be expressed as follows:
\[ D_{\text{pay floating}} = D_{\text{fixed}} - D_{\text{floating}} > 0 \]

For a pay-fixed counterparty, the duration can be expressed as follows:
\[ D_{\text{pay fixed}} = D_{\text{floating}} - D_{\text{fixed}} < 0 \]

LOS 28.c
Cash flow risk, uncertainty regarding the size of cash flows, is a concern with floating-rate instruments. Because their cash flows are reset each period according to the prevailing rate at the beginning of the period, however, their market values are subject to only minor changes.

Market value risk is a concern with fixed-rate instruments. A decline in interest rates, for example, increases the value of the liability (or pay-fixed side of a swap), thus increasing the liability of the borrower.

For individual assets and liabilities, the tradeoff is between the market value risk associated with fixed rates and the cash flow risk associated with floating rates.
LOS 28.d
The duration of the portfolio plus a swap position (i.e., the target duration) is calculated as:

\[ V_p(MD_T) = V_p(MD_p) + NP(MD_{\text{swap}}) \]

where:
- \( V_p \) = original value of the portfolio
- \( MD_i \) = modified duration \( i \) (\( i = \text{swap, target, portfolio without swap} \))
- \( NP \) = notional principal of the swap

Usually, the manager selects a swap of a certain maturity which determines the modified duration of the swap, \( MD_{\text{swap}} \). He then selects the \( NP \) that will achieve the desired \( MD_T \). Rearranging, we can solve for the amount of \( NP \) necessary to achieve the target duration as:

\[ NP = \left( \frac{V_p(MD_T - MD_p)}{MD_{\text{swap}}} \right) \]

LOS 28.e
Borrowing in a foreign country via a foreign bank may be difficult, and the interest rates may be high. A U.S. firm that wishes to initiate a project in a foreign country, say Korea, might not have the contacts necessary to borrow Korean currency (the won) cheaply. A Korean counterparty may exist that would like to borrow dollars to invest in the United States.

The U.S. firm borrows in the United States because it has established relationships with banks in the United States. It swaps the principal (borrowed dollars) with the Korean counterparty for the won, which the Korean firm borrowed in Korea.

LOS 28.f
Follow these steps in determining the appropriate swap:
1. Divide the foreign cash flow received by the foreign interest rate to determine the corresponding foreign-denominated notional principal (NP).
   a. This is the foreign NP that would have produced the foreign cash flow at the given foreign interest rate.
2. Using the current exchange rate, convert the foreign NP into the corresponding domestic NP.
3. Enter a swap with this NP.
   a. Pay the foreign cash flows received on the assets and receive the equivalent domestic amount.
   b. The amount of each domestic cash flow is determined by multiplying the domestic interest rate by the domestic NP.
LOS 28.g
A manager can swap all or part of the return on a portfolio for the return on a domestic equity index, the return on a foreign index, or the return on a fixed-income index. A manager desiring an exposure to foreign equities equivalent to 15% of the existing portfolio, for example, could enter a swap with a foreign NP equivalent to that amount. The manager pays the swap dealer the return on that portion of the portfolio and receives the return on the foreign equity index equivalent to an investment in the amount of the notional principal.

LOS 28.h
An interest rate swaption is an option on a swap where one counterparty (buyer) has paid a premium to the other counterparty (seller) for an option to choose whether the swap will actually go into effect on some future date. Swaptions can be either American or European in the same way as options.
1. Payer swaption: A payer swaption gives the buyer the right to be the fixed-rate payer (and floating-rate receiver) in a prespecified swap at a prespecified date. The payer swaption is almost like a protective put in that it allows the holder to pay a set fixed rate, even if rates have increased.

2. Receiver swaption: A receiver swaption gives the buyer the right to be the fixed-rate receiver (and floating-rate payer) at some future date. The receiver swaption is the reverse of the payer swaption. In this case, the holder must expect rates to fall, and the swap ensures receipt of a higher fixed rate while paying a lower floating rate.

The key point is that the terms of the underlying swap and the swap fixed rate (SFR) are negotiated and set at the purchase of the swaption. The purchaser of the swaption can wait and see if subsequent market moves make that SFR attractive or not and then decide whether to turn on the underlying swap.
CONCEPT CHECKERS

1. Which of the following would best transform a floating-rate liability to a fixed-rate liability? Enter into a contract to:
   A. pay fixed on an interest rate swap.
   B. receive floating on a currency swap.
   C. pay floating on an equity swap.

2. Which of the following statements most accurately describes the rights of the counterparties in a swaption structure?
   A. The holder of a receiver swaption has the right to enter a swap agreement as the fixed-rate receiver.
   B. The holder of a payer swaption has the right to enter a swap agreement as the fixed-rate receiver.
   C. The seller of a payer swaption has the right to enter a swap agreement as the fixed-rate payer.

3. A firm has most of its liabilities in the form of floating-rate notes with a maturity of two years and quarterly reset. The firm is not concerned with interest rate movements over the next four quarters but is concerned with potential movements after that. Which of the following strategies will allow the firm to hedge the expected change in interest rates?
   A. Enter into a 2-year, quarterly pay-fixed, receive-floating swap.
   B. Buy a swaption that allows the firm to be the fixed-rate payer upon exercise. In other words, go long a payer swaption with a 1-year maturity.
   C. Buy a swaption that allows the firm to be the floating-rate payer upon exercise. In other words, go short a payer swaption with a 1-year maturity.

4. A firm issues fixed-rate bonds and simultaneously becomes a fixed-rate receiver counterparty in a corresponding plain vanilla interest rate swap. Which of the following best describes the subsequent, effective periodic interest payments of the firm? (SFR = swap fixed rate)
   A. SFR – [LIBOR – (fixed rate on debt)].
   B. LIBOR – [(fixed rate on debt) – SFR].
   C. LIBOR – [SFR – (fixed rate on debt)].

5. For a plain vanilla interest rate swap, a decrease in interest rates will most likely:
   A. increase the value of the pay-fixed side of the swap.
   B. decrease the value of the pay-fixed side of the swap.
   C. leave the value of the pay-floating side unchanged.

6. A common reason for two potential borrowers in different countries to enter into a currency swap is to:
   A. borrow cheap domestic and swap for foreign to reduce borrowing costs.
   B. borrow cheap foreign and swap for domestic to reduce borrowing costs.
   C. speculate on interest rate moves.
7. A firm has an $8 million portfolio of large-cap stocks. The firm enters into an equity swap to pay a return based on the DJIA and receive a return based on the Russell 2000. To achieve an effective 60/40 mix of large-cap to small-cap exposure, the notional principal of the swap should be:
   A. $6.0 million.
   B. $4.8 million.
   C. $3.2 million.

8. For a pay-floating counterparty, the duration of the swap will generally be:
   A. less than the duration of the fixed-rate payments.
   B. equal to the duration of the fixed-rate payments.
   C. greater than the duration of the fixed-rate payments.

9. A firm will be receiving a semiannual cash flow of €20 million. The swap rates in the United States and Europe are 4.0% and 4.6%, respectively. The current exchange rate is €1.2/$. Identify the appropriate swap needed to convert the periodic euro cash flows to dollars.

10. A manager of a $40 million dollar fixed-income portfolio with a duration of 4.6 wants to lower the duration to 3. The manager chooses a swap with a net duration of 2. Determine the notional principal that the manager should choose for the swap to achieve the target duration.

11. You are the treasurer of a company with a 4-year, $20 million FRN outstanding at LIBOR. You are concerned about rising interest rates in the short term and would like to refinance at a fixed rate for the next two years. A swap dealer arranges a 2-year plain vanilla interest rate swap with annual payments in which you pay a fixed rate of 8.1% and receive LIBOR. The counterparty receives 7.9% and pays LIBOR. Assume that the counterparty has a $20 million fixed-rate debt outstanding at 8%. One-year LIBOR is currently 7%. Diagram and compute each party’s net borrowing cost and first-year cash flows.

For more questions related to this topic review, log in to your Schweser online account and launch SchweserPro™ QBank; and for video instruction covering each LOS in this topic review, log in to your Schweser online account and launch the OnDemand video lectures, if you have purchased these products.
1. A  Pay fixed means receive floating. The floating receipt on the interest swap will cover the floating payments on the liability, leaving a net pay fixed position overall. Using a currency swap introduces another currency and is not appropriate, even though receive floating is part of a correct solution. Pay floating is wrong, as is introducing equity returns into the situation.

2. A  The holder of a receiver swaption has the right to enter a swap agreement as the fixed-rate receiver.

3. B  The firm is paying floating now but may want to lock in a fixed rate of interest if interest rates rise one year from now. Hence, buy a swaption that allows the firm to be the fixed-rate payer upon exercise. In other words, go long a payer swaption with a 1-year maturity.

4. C  A swap diagram is a good way to solve this, as well as knowing swap terminology. The question asks for the net payment.

On the debt the firm is paying:
   fixed rate on debt

On the swap the firm is receiving the swap fixed rate,
   a reduction in payment: –SFR
   and paying floating: LIBOR

This is a net payment of:
   fixed rate on debt – SFR + LIBOR

Looking at the answer choices, this can also be expressed mathematically as:
   LIBOR – [SFR – (fixed rate on debt)]

5. B  Choice C is not correct because changes in rates affect both sides of the swap, and choice B best describes the result from a decrease in rates. The pay-fixed side of the swap will be paying an amount greater than the SFRs of newly issued swaps.

6. A  A domestic borrower may be able to borrow at, say, 6% and swap the principal for a foreign currency. The domestic borrower will pay the counterparty the interest on the foreign currency received. This will presumably be lower than the rate the domestic borrower would have to pay if he had borrowed directly from a foreign bank. The foreign counterparty pays the interest on the domestic loan, which is presumably lower than that it would pay if it borrowed directly from a domestic bank.

7. C  The notional principal should be 40% of the portfolio’s value.

8. A  Although most of the duration is associated with the fixed payments, the next floating payment is predetermined. Therefore, the duration of a quarterly-reset swap might be the duration of the fixed payments minus 0.125 (0.25 / 2 = 0.125).

9. For the euros, the NP = 20,000,000 / (0.046 / 2) = €869,565,217. The corresponding dollar amount is $724,637,681 = €869,565,217 / 1.2. Using these values for the swap, the firm will give the swap dealer €20,000,000 every six months over the maturity of the swap for $724,637,681(0.04 / 2) = $14,492,754.
10. From the given information, we have:
   \[ V = 40,000,000 \]
   \[ MD_V = 4.6 \]
   \[ MD_{\text{swap}} = 2.0 \]
   \[ MD_T = 3.0 \]
   \[ NP = 40,000,000 \times \left( \frac{3.0 - 4.6}{-2} \right) = 32,000,000 \]

   The manager should take a receive-floating/pay-fixed position in the swap with a $32,000,000 notional principal.

11. The box and arrow diagram is shown below:

   ![Diagram](image)

   Your net borrowing cost is:
   \[ (\text{LIBOR} - \text{LIBOR}) + 0.081 = 0.081 = 8.1\% \]

   The counterparty’s net borrowing cost is:
   \[ (0.080 - 0.079) + \text{LIBOR} = \text{LIBOR} + 0.001 = \text{LIBOR} + 0.1\% \]

   The swap dealer’s spread is:
   \[ 0.002 = 0.20\% = 20 \text{ basis points} = (0.081 - 0.079) + (\text{LIBOR} - \text{LIBOR}) \]

   At the end of the first year, assuming LIBOR is 7%, your fixed-rate payment under the swap is:
   \[ \text{fixed-rate payment} = (0.081 - 0.07)(20,000,000) = 220,000 \]

   Your total interest costs equal the LIBOR-based interest payments plus the swap payment:
   \[ 20,000,000(0.07) + 220,000 = 1,620,000 \]

   At the end of the first year, the counterparty’s fixed-rate receipt under the swap is:
   \[ (\text{fixed-rate receipt}) = (0.079 - 0.07)(20,000,000) = 180,000 \]

   The counterparty’s total interest costs equal the 8% interest payment on their outstanding fixed-rate debt minus the swap payment:
   \[ 20,000,000(0.08) - 180,000 = 1,420,000 \]

   The cash flows to the swap dealer are:
   \[ 220,000 - 180,000 = (20,000,000 \times 0.002) = 40,000 \]

   Everybody is happy. You’ve converted floating-rate debt to fixed-rate debt, your counterparty has converted fixed-rate debt to floating-rate debt, and the swap dealer has made $40,000 without being exposed to interest rate risk.